Chapter 10
Binomial Distributions

Lesson 10-4 (pp. 637–644)

1. The total height of the bars in a histogram for a probability distribution is the sum of all of the probabilities, which is 1.

2. false; The mean of the random variable is the probability weighted mean of the possible outcomes. The mean can differ from the possible outcomes.

3. The probabilities do not add to 1 so this is not a probability distribution.

4. This is a probability distribution. The mean is 0(1) + 0(2) + 1(3) + 0(4) = 3.

5. This is not a probability distribution because the probabilities do not add to 1.

6. This is a probability distribution. The mean is 0.18(1) + 0.27(2) + 0.45(3) + 0.10(4) = 2.47.

7. a. The random variable is the number of days of incubation time.

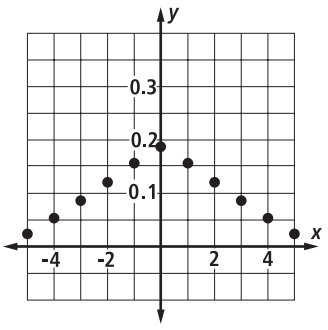
b. Find the weighted sum of the random variable.

$$1\left(\tfrac{1}{14}\right)+2\left(\tfrac{3}{28}\right)+3\left(\tfrac{5}{21}\right)+4\left(\tfrac{1}{7}\right)$$
$$+5\left(\tfrac{1}{3}\right)+6\left(\tfrac{1}{14}\right)+7\left(\tfrac{1}{28}\right)\approx 3.92 \text{ days}$$

c. The mode is the value with the highest probability, which is 5.

8. a. See page 292.

b.



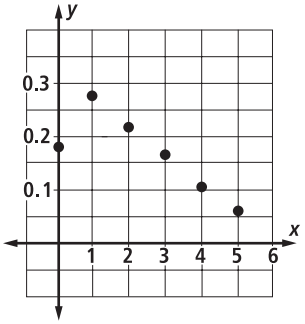
c. The mean is the x-value times the probability. The graph is symmetric across the y-axis so the probability for x equals the probability for $-x$. This means that $px + p(-x) = 0$ so the mean is 0.

9. a. To find the variance we can find the mean of x^2 and subtract the mean squared. This gives

$$\left[0\left(\tfrac{36}{52}\right)+1\left(\tfrac{4}{52}\right)+4\left(\tfrac{4}{52}\right)+9\left(\tfrac{4}{52}\right)+16\left(\tfrac{4}{52}\right)\right]-\left(\tfrac{10}{13}\right)^2\approx 3.31.$$

b. The standard deviation is the square root of the variance. This gives about 1.82.

10. a.



b. Multiply the difference by the relative frequency and add them.

$$0\left(\tfrac{62}{360}\right)+1\left(\tfrac{98}{360}\right)+2\left(\tfrac{77}{360}\right)+$$
$$3\left(\tfrac{60}{360}\right)+4\left(\tfrac{38}{360}\right)+5\left(\tfrac{25}{360}\right)\approx 1.97.$$

11. a.

Difference	0	1	2	3	4	5
$P(x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

b. Multiply the difference by the probability and add them.

$$0\left(\tfrac{1}{6}\right)+1\left(\tfrac{5}{18}\right)+2\left(\tfrac{2}{9}\right)+3\left(\tfrac{1}{6}\right)+$$
$$4\left(\tfrac{1}{9}\right)+5\left(\tfrac{1}{18}\right)\approx 1.94$$

These are different because 1.97 was the mean of the relative frequencies, and the relative frequencies only approximate the actual probabilities.

c. To find the variance, find the mean of x^2 and subtract the mean squared.
$\left[0\left(\frac{1}{6}\right)+1\left(\frac{5}{18}\right)+4\left(\frac{2}{9}\right)+9\left(\frac{1}{6}\right)\right.$
$\left.+16\left(\frac{1}{9}\right)+25\left(\frac{1}{18}\right)\right]-1.94^2\approx 2.05$

d. The standard deviation is the square root of the variance; about 1.43.

12. a. It is the amount you make if you buy a ticket and don't win (in total you lose a dollar).

b. The probability of winning nothing is 1 – (probability of winning something). The chance of winning something is $\frac{3}{125}$, so the probability of winning nothing is $1-\frac{3}{125}=\frac{122}{125}$.

c. The mode is -1, or winning nothing because it has the highest probability.

d. Multiply the probability by the value of the random variable and add them.
$49\left(\frac{1}{125}\right)+29\left(\frac{1}{125}\right)+19\left(\frac{1}{125}\right)$
$+(-1)\left(\frac{122}{125}\right)=-\frac{1}{5}$

13. a. First change the number to a relative frequency by dividing by the total number of animals counted. Then multiply the age at death by the relative frequency and add them.
$1\left(\frac{30}{550}\right)+2\left(\frac{86}{550}\right)+3\left(\frac{132}{550}\right)+4\left(\frac{173}{550}\right)+$
$5\left(\frac{77}{550}\right)+6\left(\frac{40}{550}\right)+7\left(\frac{10}{550}\right)+8\left(\frac{2}{550}\right)\approx$
3.64 years

b. Find the mean of x^2 and subtract the mean squared.
$\left[1^2\left(\frac{30}{550}\right)+2^2\left(\frac{86}{550}\right)+3^2\left(\frac{132}{550}\right)\right.$
$+4^2\left(\frac{173}{550}\right)+5^2\left(\frac{77}{550}\right)+6^2\left(\frac{40}{550}\right)$
$\left.+7^2\left(\frac{10}{550}\right)+8^2\left(\frac{2}{550}\right)\right]$
$-3.64^2\approx 1.88$ years.

c. Take the square root of the variance; about 1.37 years.

8. a.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

14. A and D could be probability distributions because their probabilities (on the *y*-axis) add to 1.

15. a. Row 7 of Pascal's triangle is 1, 7, 21, 35, 35, 21, 7, 1. All of these are divisible by 7 except the first and the last.

b. Row 9 of Pascal's triangle is 1, 9, 36, 84, 126, 126, 84, 36, 9, 1; 84 is not divisible by 9.

c. ${}_nC_r=\frac{n!}{r!(n-r)!}$; We know that n is prime and r and $(n-r)$ are smaller than n, so any part of $r!$ multiplied times any part of $(n-r)!$ will never equal (and thus never cancel) n from the numerator.

16. The 17th term is given by ${}_{43}C_{16}=265,182,149,218$, and the 21st term is given by ${}_{43}C_{20}=960,566,918,220$.

17. The order of the group of 3 does not matter so we use combinations. This is given by ${}_6C_3=20$.

18. The order here matters so we use permutations. Use the Multiplication Counting Principle.
10(9)(8)(7)(6)(5)(4)(3)(2) = 3,628,800

19. a. There are two possibilities, heads and tails. The chi-square statistic can be given as
$\frac{(9-5)^2}{5}+\frac{(1-5)^2}{5}=\frac{32}{5}=6.4$.

b.

χ²GOF {9,1},{5,5},1: *stat.results*

"Title"	"χ² GOF"
"χ²"	6.4
"PVal"	0.011412
"df"	1.
"CompList"	"{...}"

c. There is insignificant evidence to reject the hypothesis that the coin is fair because 0.011 is greater than the significance level of 0.01.

20. The three math books must be together but the order is not defined, so we must find out how many different ways the three can be ordered. This is ${}_3P_3 = 6$. Using the three math books as a group (the same as 1 book) we need to find the number of ways that these 5 books can be ordered. This is ${}_5P_5 = 120$. The total number of ways is ${}_3P_3 \bullet {}_5P_5 = 720$.

21. a. Answers vary. Sample: The life expectancy for a 20-year-old male in the United States is 56.2 years, for a total lifetime of 76.2 years.

b. Life expectancies are determined by measuring the probability of surviving from one year to the next for all relevant ages. These are used to create a "life table." The life expectancy is the mean of the random variable, in this case life expectancy.