g. The elevator will take another 36 ft to slow down and stop. So the deceleration should start where the elevator is 564 ft up, about the 47th floor (from part h, one floor = 12 ft).

h. The elevator takes a total of 12 s to accelerate and decelerate. During these intervals it travels a total of 72 ft, leaving 528 ft for the constant velocity portion. At 12 ft/s, this part of the trip will take 44 s. Thus, the total trip takes 56 s.

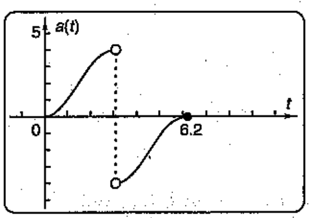
i. The elevator must start to decelerate halfway through the trip, where $s(t) = 6$ ft. Solving

$$\int_0^b \left(2t - \frac{6}{\pi}\sin\frac{\pi}{3}t\right) dt = 6$$

numerically for b gives $b \approx 3.1043\ldots \approx$ 3.1 s.

$a(3.1043\ldots) = 3.9880\ldots \approx 4.0$ ft/s^2

By symmetry, the deceleration process must start at this time, meaning the acceleration jumps to $-3.9880\ldots$ ft/s^2. The graph looks like this:



Thus, the passengers get a large jerk at the midpoint of the trip.

One way to remedy the problem is to reduce the acceleration so that the elevator goes only 6 ft instead of 36 ft in the first 6 seconds. That is,

$$a(t) = \frac{1}{3} - \frac{1}{3}\cos\frac{\pi}{3}t$$

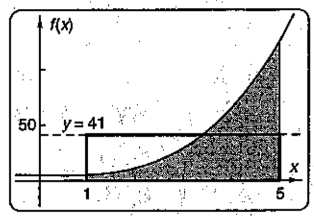
You may think of other ways.

Problem Set 10-3

Q1. 50 mi/h	Q2. 30 mi
Q3. 20 min	Q4. 2π
Q5. No local maximum	Q6. 1.5
Q7. $f(x) = 16$ (at $x = 1$)	Q8. infinite
Q9. Mean value theorem	Q10. D

1. a. $y_{av} = \dfrac{1}{4}\displaystyle\int_1^5 (x^3 - x + 5)\, dx = \dfrac{1}{4}(164) = 41$

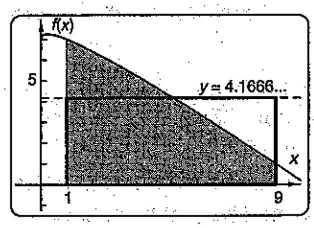
b. The rectangle has the same area as the shaded region.



c. $41 = c^3 - c + 5$
$c = 3.4028\ldots$, which is in [1, 5].

2. a. $y_{av} = \dfrac{1}{8}\displaystyle\int_1^9 (x^{1/2} - x + 7)\, dx = 4\dfrac{1}{6}$

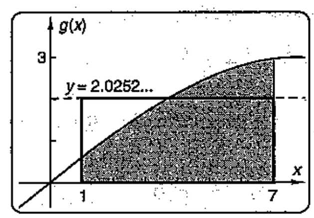
b. The rectangle has the same area as the shaded region.



c. $4\dfrac{1}{6} = c^{1/2} - c + 7$
$c = 5.0892\ldots$, which is in [1, 9].

3. a. $y_{av} = \dfrac{1}{6}\displaystyle\int_1^7 3\sin 0.2x\, dx = 2.0252\ldots$

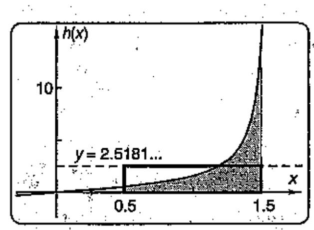
b. The rectangle has the same area as the shaded region.



c. $2.0252\ldots = 3\sin 0.2c$
$c = 3.7053\ldots$, which is in [1, 7].

4. a. $y_{av} = \dfrac{1}{1}\displaystyle\int_{0.5}^{1.5} \tan x\, dx = 2.5181\ldots$

b. The rectangle has the same area as the shaded region.

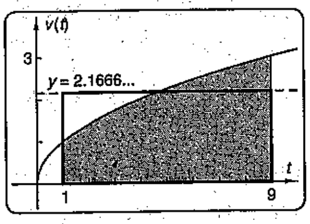


c. $2.5181\ldots = \tan c$

$c = 1.1927\ldots$, which is in [0.5, 1.5].

5. a. $y_{av} = \frac{1}{8}\int_1^9 \sqrt{t}\,dt = 2\frac{1}{6}$

b. The rectangle has the same area as the shaded region.



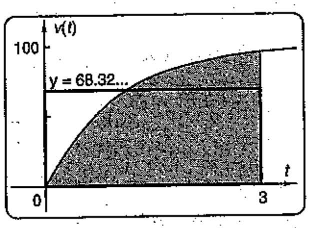
c. $2\frac{1}{6} = \sqrt{c}$

$c = 4\frac{25}{36}$, which is in [1, 9].

6. a. $y_{av} = \frac{1}{3}\int_0^3 100(1-e^{-t})\,dt = \frac{100}{3}(2+e^{-3})$

$= 68.3262\ldots$

b. The rectangle has the same area as the shaded region.



c. $68.3262\ldots = 100(1 - e^{-c})$

$c = 1.1496\ldots$, which is in [0, 3].

7. $y_{av} = \frac{1}{k}\int_0^k ax^2\,dx = \frac{1}{3}ak^2$

8. $y_{av} = \frac{1}{k}\int_0^k ax^3\,dx = \frac{1}{4}ak^3$

9. $y_{av} = \frac{1}{k}\int_0^k ae^x\,dx = \frac{1}{k}a(e^k - 1)$

10. $y_{av} = \frac{1}{k}\int_0^k \tan x\,dx = \frac{1}{k}\ln|\sec k|$

11. $a(t) = 6t^{-1/2}$

$v(t) = 12t^{1/2} + C$; $v(0) = 60 \Rightarrow C = 60$

$v(t) = 12t^{1/2} + 60$

$s(t) = 8t^{3/2} + 60t + s_0$

$v(25) = 120$ ft/s

Displacement $= s(25) - s(0) = 2500$ ft

$v_{av} = 2500/25 = 100$ ft/s

12. The general equation of a parabola with vertex (h, k) is $v - k = a(t-h)^2$. Vertex is at $(t, v) = (2, 50)$, so

$v - 50 = a(t-2)^2$. $v = 30$ when $t = 0$, so

$-20 = a(-2)^2 \Rightarrow a = -5$.

$v = 50 - 5(t-2)^2$

$v_{av} = \frac{1}{4}\int_0^4 [50 - 5(t-2)^2]\,dt = 43\frac{1}{3}$ mi/h

This is just $13\frac{1}{3}$ mi/h above the speed limit.

If Ida wins her appeal, her fine will be

$7 \cdot 13\frac{1}{3} = \$93\frac{1}{3} \approx \$93.33$, which is \$46.67 less than what she now faces.

13. Consider an object with constant acceleration a, for a time interval $[t_0, t_1]$.

$v(t) = \int a\,dt = at + C$

At $t = t_0$, $v(t) = v_0 \Rightarrow v_0 = at_0 + C \Rightarrow$

$C = v_0 - at_0$.

$\therefore v(t) = at + v_0 - at_0 = v_0 + a(t - t_0)$

$$v_{av} = \frac{\int_{t_0}^{t_1}[v_0 + a(t - t_0)]\,dt}{t_1 - t_0}$$

$$= \frac{1}{t_1 - t_0}\left[v_0t_1 + \frac{1}{2}a(t_1 - t_0)^2 - v_0t_0 - \frac{1}{2}a(t_0 - t_0)^2\right]$$

$$= v_0 + \frac{1}{2}a(t_1 - t_0)$$

The average of v_0 and v_1 is

$$\frac{1}{2}(v_0 + v_1) = \frac{1}{2}[v_0 + v_0 + a(t_1 - t_0)]$$

$$= v_0 + \frac{1}{2}a(t_1 - t_0)$$

$\therefore v_{av}$ = the average of v_0 and v_1, Q.E.D.

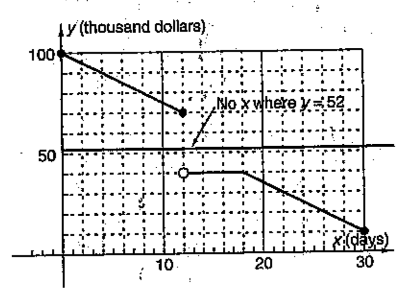
14. Counterexample: In Problem 11, the car's acceleration is $a = 6/\sqrt{t}$. The initial velocity is $v(0) = 60$ ft/s; the final velocity after 25 seconds is $v(25) = 120$ ft/s; and the average velocity is $v_{av} = 100$ ft/s. But the average of the initial and final velocities is $\frac{1}{2}[v(0) + v(25)] = 90 \text{ ft/s} \neq v_{av}$.

15. a. Integral = area = 12(100 + 70)/2 + 6(40) + 12(40 + 10)/2 = 1560

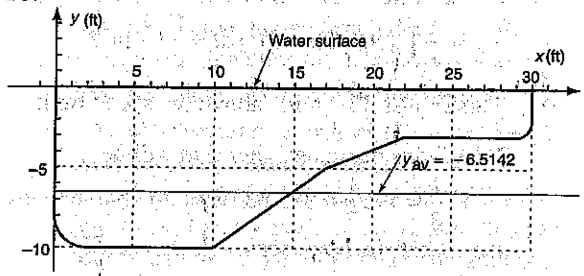
$y_{av} = 1560/30 = 52$, or \$52,000

Cost of inventory = 0.50(52000)/100 = \$260.00

b. At $x = 12$, they may have had a single, large sale, dropping the inventory from \$70,000 to \$40,000. There is no day on which the inventory is worth \$52,000.



16.



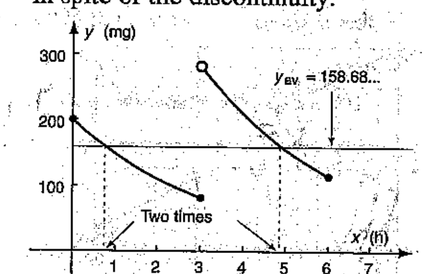
Integral = −(area of 4 rectangles, 2 trapezoids, and 2 quarter-circles)
$2(-8) + 8(-10) + 7(-3) + 1(-2) + 7[-10 + (-5)]/2 + 5[-5 + (-3)]/2 - \pi(2^2) - \pi(1)^2/4 = -195.4269...$
$y_{av} = -195.4269.../30 = -6.5142...$, or about 6.51 feet deep.
The volume would equal 6.5142... times the area of the horizontal cross section times the number of gallons in a cubic foot.

17. Integral $\approx 3(16/2 + 15 + 15 + 17/2) + 2(17 + 20)/2 + 1(20 + 14)/2 + 3(14/2 + 10 + 9 + 8 + 9/2) = 139.5 + 37 + 17 + 115.5 = 309$
$y_{av} = 309/24 = 12.875 \approx 12.9°C$
The average of the high and low temperatures is $(20 + 8)/2 = 14°C$, which is higher than the actual average. Averaging high and low temperatures is easier than finding the average by calculus, but the latter is more realistic for such applications as determining heating and air conditioning needs.

18. a. At $x = 3$, $y = 81.3139... \approx 81.3$ mg.
$$y_{av} = \frac{1}{3}\int_0^3 200e^{-0.3x}\,dx = \frac{1}{3}(395.6202...)$$
$$= 131.8734... \approx 131.9 \text{ mg}$$

b. $k = 81.3139...$, so the equation is $y = 281.3139...e^{-0.3(x-3)}$.
$$y_{av} = \frac{1}{6}\left[395.6202... + \int_3^6 281.3139...e^{-0.3(x-3)}\,dx\right]$$
$$= \frac{1}{6}(395.6202... + 556.4674...) = 158.6812...$$
$$\approx 158.7 \text{ mg}$$

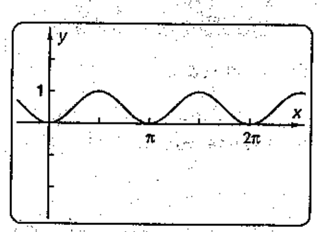
c. As the graph shows, there are two times in [0, 6] at which there are 158.7 mg. So the conclusion of the mean value theorem is true, in spite of the discontinuity.



19. $v = A \sin 120\pi t$ and $y = |A \sin 120\pi t|$
$$y_{av} = \frac{1}{1/60}\int_0^{1/60} |A\sin 120\pi t|\,dt$$
$$= 60\int_0^{1/120} A\sin 120\pi t\,dt - 60\int_{1/120}^{1/60} A\sin 120\pi t\,dt$$
$$= -\frac{A}{2\pi}\cos 120\pi t\,\Big|_0^{1/120} + \frac{A}{2\pi}\cos 120\pi t\,\Big|_{1/120}^{1/60}$$
$$= \frac{A}{2\pi}(-\cos\pi + \cos 0 + \cos 2\pi - \cos\pi) = \frac{2A}{\pi}$$
If $y_{av} = 110$, then $\frac{2A}{\pi} = 110 \Rightarrow A = 55\pi$
$= 172.78...$ V.
The average value of one arc of $y = \sin x$ is $\frac{1}{\pi - 0}\int_0^{\pi} \sin x\,dx = \frac{2}{\pi}$, and $y = \sin x$ has a maximum value of 1. A horizontal stretch does not affect the average value. Write a proportion to find the maximum of a sinusoidal curve with an average value of 110. $\frac{2/\pi}{1} = \frac{110}{m}$, so $m = 55\pi$.

20. a. $d = k \sin x$
$$d_{av}^2 = \frac{1}{2\pi}\int_0^{2\pi} k^2 \sin^2 x\,dx$$
$$= \frac{k^2}{2\pi}\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right)\Big|_0^{2\pi}$$
$$= \frac{k^2}{2\pi}(\pi - 0 - 0 + 0) = \frac{k^2}{2}$$
$\therefore$ rms $= k/\sqrt{2} = 0.7071...k$

b. $\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$
Thus, $\sin^2 x$ is a sinusoid.



c. By symmetry across the line $y = \frac{1}{2}$, the average of $y = \frac{1}{2} - \frac{1}{2}\cos 2x$ (and hence $y = \sin^2 x$) over $[0, 2\pi]$ is $\frac{1}{2}$. Thus, the average of $y = k^2 \sin^2 x$ is $\frac{1}{2}k^2$.
$\therefore$ rms $= k/\sqrt{2}$, as in part a.