15. a. Simpson's rule will give a more accurate answer because the function $y = \sin x$ is approximated better by quadratic functions than by straight lines.

b. $S_4 = (1/3)(\pi/4)[\sin 0 + 4 \sin (\pi/4) + 2 \sin (\pi/2) + 4 \sin (3\pi/4) + \sin \pi] = 2.0045\ldots$

$T_4 = (1/2)(\pi/4)[\sin 0 + 2 \sin (\pi/4) + 2 \sin (\pi/2) + 2 \sin (3\pi/4) + \sin \pi] = 1.8961\ldots$

$$\int_0^{\pi} \sin x\, dx = -\cos x \Big|_0^{\pi} = \cos \pi - \cos 0 = 2$$

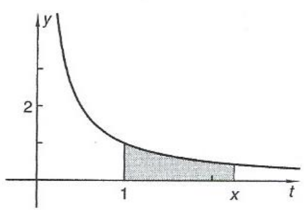
Simpson's rule does give a better approximation of the integral because S_4 is closer to 2 than is T_4.

16. Programs will vary depending on the type of grapher used. See the program in the Programs for Graphing Calculators section of the *Instructor's Resource Book.*

17. Using a Simpson's rule program, the mass of the spleen is 171.6 cm^3.

18. Enter $\text{Y1} = \frac{2}{\sqrt{\pi}} e^{-x^2}$. A Simpson's rule program gives $S_{50} = 0.5204998781\ldots$ and $S_{100} = 0.5204998778\ldots$. There is little difference between the two estimations, and both are close to the tabulated value.

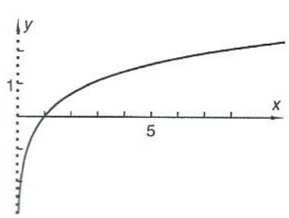
19. a.



As x varies, the area beneath the curve $y = 1/t$ from $t = 1$ to $t = x$ varies also.

b. Using the power formula on $\int t^{-1}\, dt$ gives $\frac{t^0}{0}$. Division by zero is undefined, so this approach does not work.

c. Graph $\text{Y1} = \text{fnint}(x^{-1}, x, 1, x)$. (Entries may be different for different calculators.) The graph looks like $y = \ln x$. The value of $f(x)$ is negative for $x < 1$ because for these values the lower limit of integration is larger than the upper limit, resulting in negative values for dx.



d. $f(2) = 0.6931\ldots$

$f(3) = 1.0986\ldots$

$f(6) = 1.7917\ldots$

$f(2) + f(3) = f(2 \cdot 3)$. This is a property of logarithmic functions.

Problem Set 5-11

Review Problems

R0. Answers will vary.

R1. a. The width of each region is 4. So $T_3 = (4/2)[v(4) + 2v(8) + 2v(12) + v(16)] = 2[22 + 2(26.9705\ldots) + 2(30.7846\ldots) + 34] = 343.0206\ldots$. T_3 underestimates the integral because $v(t)$ is concave down, so trapezoids are inscribed under the curve.

b. $R_3 = 4[v(6) + v(10) + v(14)] = 4(24.6969\ldots + 28.9736\ldots + 32.4499\ldots) = 344.4821\ldots$

This Riemann sum is close to the trapezoidal-rule sum.

c. $T_{50} = 343.9964\ldots$, and $T_{100} = 343.9991\ldots$ Conjecture: The exact value of the integral is 344.

d. $g(t) = 10t + 4t^{1.5}$

$g(16) - g(4) = 344$

This is the value the trapezoidal-rule sums are approaching.

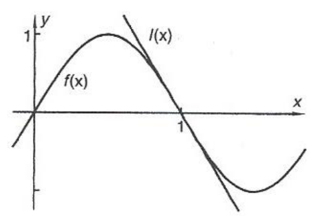
R2. a. The slope of the linear function is the same as the slope of the curve at $x = 1$. So the slope is

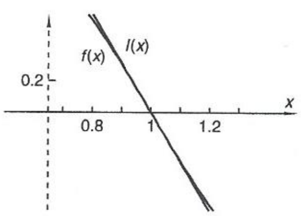
$f(x) = \sin \pi x \Rightarrow f'(x) = \pi \cos \pi x \Rightarrow$

$f'(1) = \pi \cos \pi = -\pi$

At $x = 1$, $y = \sin \pi = 0$

$y - 0 = -\pi (x - 1) \Rightarrow l(x) = -\pi x + \pi$





As you zoom in, you see that $f(x)$ is very close to the line $l(x)$ for values near $x = 1$.

For $x = 1.1$, the error is $\sin[\pi(1.1)] - [-\pi(1.1) + \pi] = 0.0051...$

For $x = 1.001$, the error is $\sin[\pi(1.001)] - [-\pi(1.001) + \pi] = 5.1677... \times 10^{-9}$.

b. i. $y = \csc^5 2x \Rightarrow dy = -10 \csc^5 2x \cot 2x\, dx$

ii. $y = x^5/5 - x^{-3}/3 \Rightarrow dy = (x^4 + x^{-4})\, dx$

iii. $y = (7 - 3x)^4 \Rightarrow dy = -12(7 - 3x)^3\, dx$

iv. $y = 5e^{-0.3x} \Rightarrow dy = -1.5e^{-0.3x}\, dx$

v. $y = \ln(2x)^4 \Rightarrow dy = \dfrac{1}{(2x)^4} \cdot 4(2x)^3 \cdot 2\, dx$

$= 4/x\, dx$; or $y = \ln(2x)^4 = 4\ln(2x) \Rightarrow$

$dy = 4 \cdot \dfrac{1}{2x} \cdot 2dx = 4/x\, dx$

c. i. $dy = \sec x \tan x\, dx \Rightarrow y = \sec x + C$

ii. $dy = (3x+7)^5\, dx \Rightarrow y = \dfrac{1}{18}(3x+7)^6 + C$

iii. $dy = 5\, dx \Rightarrow y = 5x + C$

iv. $dy = 0.2e^{-0.2x} \Rightarrow y = -e^{-0.2x} + C$

v. $dy = 6^x\, dx \Rightarrow y = \dfrac{6^x}{\ln 6} + C$

d. i. $y = (2x + 5)^{1/2} \Rightarrow dy = (2x + 5)^{-1/2}\, dx$

ii. $x = 10$ and $dx = 0.3 \Rightarrow$

$dy = 25^{-1/2} \cdot 0.3 = 0.06$

iii. $\Delta y = (2 \cdot 10.3 + 5)^{1/2} - (2 \cdot 10 + 5)^{1/2}$

$= 0.059644...$

iv. 0.06 is close to 0.059644... .

R3. a. See the text for the definition of *indefinite integral.*

b. i. $\int 12x^{2/3}\, dx = 7.2x^{5/3} + C$

ii. $\int \sin^6 x \cos x\, dx = \dfrac{1}{7}\sin^7 x + C$

iii. $\int (x^2 - 8x + 3)\, dx = \dfrac{1}{3}x^3 - 4x^2 + 3x + C$

iv. $\int 12e^{3x} dx = 4e^{3x} + C$

v. $\int 7^x dx = \dfrac{7^x}{\ln 7} + C$

R4. a. See the text for the definition of *integrability.*

b. See the text for the definition of *definite integral.*

c. $\displaystyle\int_{0.2}^{1.4} \sec x\, dx$

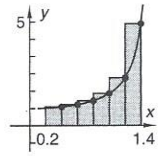
i. $U_6 = 2.845333...$

ii. $L_6 = 1.872703...$

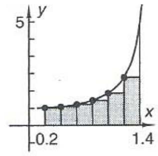
iii. $M_6 = 2.209073...$

iv. $T_6 = 2.359018...$

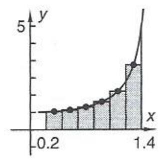
d. U_6



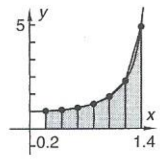
L_6



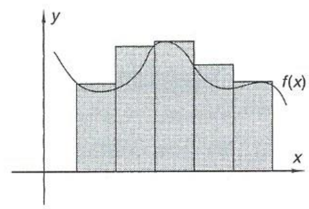
M_6



T_6



e.



R5. a. The hypothesis is the "if" part of a theorem, and the conclusion is the "then" part. (*Hypo-* means "under," and *-thesis* means "theme.")

b. $d(t) = 20 + 3\sin\dfrac{\pi}{4}t$

Average velocity $= \dfrac{d(2) - d(0)}{2 - 0} = 1.5$ m/s

Instantaneous velocity $= d'(t) = 0.75\pi \cos\dfrac{\pi}{4}t$

$d'(c) = 0.75\pi \cos\dfrac{\pi}{4}c = 1.5$

$\dfrac{\pi}{4}c = \cos^{-1}(2/\pi) = 0.880689...$

$c = 1.12132... \approx 1.12$ s

c. $g(x) = x^{4/3} - 4x^{1/3} = x^{1/3}(x - 4)$

$g(x) = 0 \Rightarrow x = 0$ or $x = 4$. Interval is $[0, 4]$.

$g'(x) = (4/3)x^{1/3} - (4/3)x^{-2/3} = (4/3)x^{-2/3}(x - 1)$

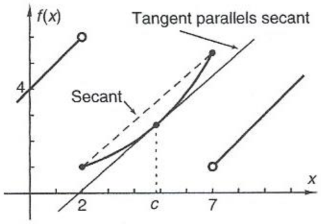
$g'(c) = 0 \Rightarrow c = 1$

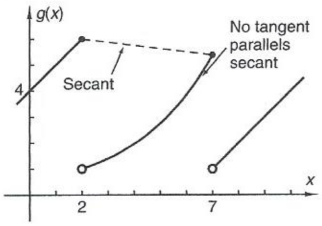
At $x = 0$, $g'(0)$ takes the form 1/0, which is infinite.

Thus, g is not differentiable at $x = 0$. However, the function need not be differentiable at the endpoints of the interval, just continuous at the endpoints and differentiable at interior points.

d. For a function to be continuous on a *closed* interval, the limit needs to equal the function value only as x approaches an endpoint from *within* the interval. This is true for function f

at both endpoints, but not true for function g at $x = 2$. The graphs show that the conclusion of the mean value theorem is true for f but not for g.

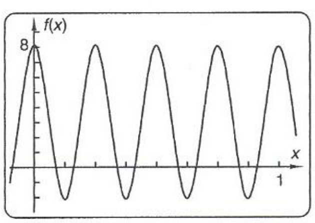




(Middle branch has the equation $y = 1.4^{(x-2)}$. Point $c = 4.4825\ldots$.)

e. g is the linear function containing the points $(a, f(a))$ and $(b, f(b))$. h is the function $h(x) = f(x) - g(x)$. Thus, $h(a) = h(b) = 0$, satisfying one hypothesis of Rolle's theorem. The other two hypotheses are satisfied because f and g are differentiable and continuous at the appropriate places, and a difference of differentiable and continuous functions also has these properties. The c in (a, b) for which $h'(c) = 0$ turns out to be the c in (a, b) for which $f'(c)$ equals the slope of the secant line, $g'(c)$, which equals $[f(b) - f(a)]/(b - a)$.

f.



Points are $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{7}{8}$.

g. If $r'(x) = s'(x)$ for all x in an interval, then $r(x) = s(x) + C$ for some constant C.

R6. a. $g(x) = \int x^{1.5}\, dx = 0.4x^{2.5} + C$

Without loss of generality, let $C = 0$.

$g'(c_1) = \dfrac{g(2) - g(1)}{2 - 1} = 1.862741\ldots$

$\therefore c_1^{1.5} = 1.862741\ldots \Rightarrow$

$c_1 = (1.862741\ldots)^{1/1.5} = 1.513915927\ldots$

Similarly, $c_2 = 2.50833898\ldots$.

$c_3 = 3.505954424\ldots$

For $\int_1^4 x^{1.5}\, dx$,

$R_3 = (1.513\ldots)^{1.5} + (2.508\ldots)^{1.5} + (3.505\ldots)^{1.5}$ $= 12.4$, which is the exact value of the integral.

b. $\int_{-1}^{3} (10 - x^2)\, dx = 10x - (1/3)x^3 \Big|_{-1}^{3}$

$= 30 - 9 - (-10) + (-1/3) = 92/3 = 30.6666\ldots$

c. $T_{100} = 30.6656$, which is close to $92/3$.

d. $M_{10} = 30.72$

$M_{100} = 30.6672$

$M_{1000} = 30.666672$

These Riemann sums are approaching $92/3$.

R7. a. i. $\int_1^5 x^{-2}\, dx = -x^{-1}\Big|_1^5 = -5^{-1} + 1^{-1} = 4/5$

ii. $\int_3^4 (x^2 + 3)^5 (x\, dx)$

$= (1/2)\int_3^4 (x^2 + 3)^5\ (2x\, dx)$

$= (1/12)(x^2 + 3)^6\Big|_3^4$

$= (1/12)(19)^6 - (1/12)(12)^6$

$= 3{,}671{,}658.08\ldots$

iii. $\int_0^{\pi} (\sin x - 5)\, dx = -\cos x - 5x\Big|_0^{\pi}$

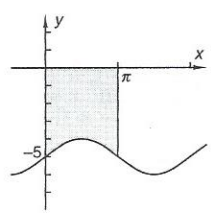
$= -\cos \pi - 5\pi + \cos 0 + 0 = 2 - 5\pi$

iv. $\int_0^{\ln 5} 4e^{2x} dx = 2e^{2x}\Big|_0^{\ln 5} = 2e^{2\ln 5} - 2e^0 = 48$

v. $\int_1^4 3^x\, dx = \dfrac{3^x}{\ln 3}\Big|_1^4 = \dfrac{3^4}{\ln 3} - \dfrac{3^1}{\ln 3} = \dfrac{78}{\ln 3}$

$= 70.9986\ldots$

b.

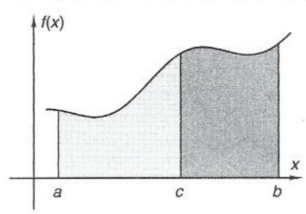


Integral is negative, because each y-value in the Riemann sum is negative.

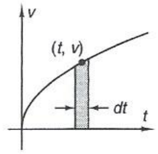
c. $\int_{-10}^{10} (4 \sin x + 6x^7 - 8x^3 + 4)\, dx = 2\int_0^{10} 4\, dx$

$= 8x\Big|_0^{10} = 80$

d. Total area = sum of two areas



R8. a.



$dy = v\,dt = 150t^{0.5}\,dt$

$y = \int_0^9 150t^{0.5}\,dt = 100t^{1.5}\Big|_0^9 = 2700$ ft

For [0, 4], $y = \int_0^4 150t^{0.5}\,dt = 800$.

For [4, 9], $y = \int_4^9 150t^{0.5}\,dt = 2700 - 800$

$= 1900$.

So $\int_0^9 v(t)\,dt = 2700 = 800 + 1900$

$= \int_0^4 v(t)\,dt + \int_4^9 v(t)\,dt$.

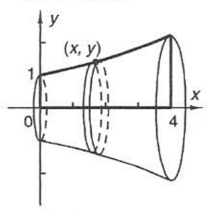
b. $dA = x\,dy$

$y = \ln x \Rightarrow x = e^y$

$dA = e^y\,dy$

$\int_0^{\ln 4} e^y\,dy = e^y\Big|_0^{\ln 4} = e^{\ln 4} - e^0 = 4 - 1 = 3$

R9. a. $y = e^{0.2x}$, from $x = 0$ to $x = 4$, is rotated about the x-axis.

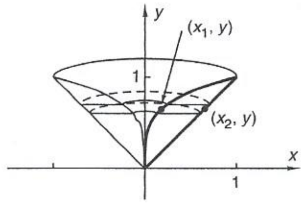


$dV = \pi y^2\,dx = \pi e^{0.4x}\,dx$

$V = \int_0^4 \pi e^{0.4x}\,dx = 2.5\pi e^{0.4x}\Big|_0^4$

$= 2.5\pi(e^{1.6} - 1) = 31.0470\ldots$

b. $y = x_1^{0.25}$ and $y = x_2$, intersecting at (0, 0) and (1, 1) in Quadrant I, is rotated about the y-axis. Only the back half of the solid is shown.



$y = x_1^{0.25} \Rightarrow x_1 = y^4$

$y = x_2 \Rightarrow x_2 = y$

$dV = \pi(x_2^2 - x_1^2)\,dy = \pi(y^2 - y^8)\,dy$

$V = \int_0^1 \pi(y^2 - y^8)\,dy = \pi\left(\frac{1}{3}y^3 - \frac{1}{9}y^9\right)\Big|_0^1$

$= \frac{2}{9}\pi = 0.6981\ldots$

c. $y = x_1 + 2 \Rightarrow x_1 = y - 2$

$y = 3x_2 - 6 \Rightarrow x_2 = \frac{1}{3}y + 2$

Graphs intersect at $y = 6$. Diameter of circular cross section is $(x_2 - x_1)$.

$dV = \pi[0.5(x_2 - x_1)]^2\,dy$

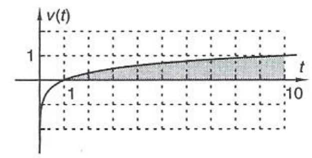
$= \frac{\pi}{4}\left[\left(\frac{1}{3}y + 2\right) - (y - 2)\right]^2 dy = \frac{\pi}{4}\left(4 - \frac{2}{3}y\right)^2 dy$

$V = \int_0^6 dV \approx 25.1327\ldots$ (exactly 8π)

The right circular cone of altitude 6 and radius 2 also has volume $\frac{1}{3}\pi \cdot 2^2 \cdot 6 = 8\pi$.

R10. a. $\int_1^{10} \log x\,dx = 6.0913\ldots$

The integral is reasonable because counting squares gives approximately 6.



b. $dW = v \cdot y \cdot dx = (1000 + 50x)(4 - 0.2x^2)\,dx$

$\int_0^3 (1000 + 50x)(4 - 0.2x^2)\,dx = 10{,}897.5$

c. $\int_3^5 v(t)\,dt = 1/3(0.2)[29 + 4(41) + 2(50) + 4(51)$

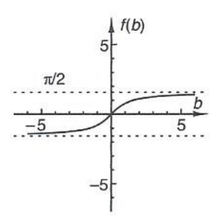
$+ 2(44) + 4(33) + 2(28) + 4(20)$

$+ 2(11) + 4(25) + 39] = 67.6$

Values of velocity are more likely to be connected by smooth curves than by straight lines, so the quadratic curves given by Simpson's rule will be a better fit than the straight lines given by the trapezoidal rule.

Concept Problems

C1. a.



b.