b. $f'(x) = 3x^2 - 14x + 9$

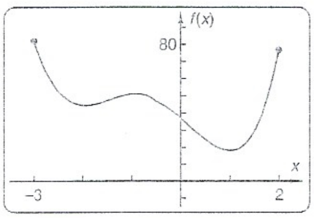
$f'(x) = 0$ at $x = \dfrac{1}{3}(7 \pm \sqrt{22}) = 3.896\ldots$ or $0.769\ldots$

$f''(x) = 6x - 14$; $f''(x) = 0$ at $x = \dfrac{7}{3} = 2.333\ldots$

c. $f''(0.769\ldots) = 6(0.769\ldots) - 14 = -9.3808\ldots < 0$, confirming local maximum.

d. Critical and inflection points occur only where f, f', or f'' is undefined (no such points exist) or is zero (all such points are found above).

35. a. $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 37$, $x \in [-3, 2]$



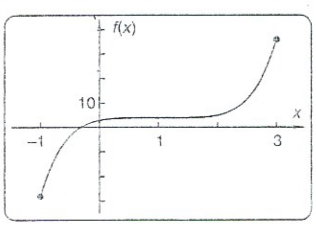
Maximum $(-3, 82)$, $(-1, 50)$, $(2, 77)$, minimum $(-2, 45)$, $(1, 18)$, points of inflection $(-1.5, 45.7)$, $(0.2, 32.0)$
Global maximum at $(-3, 82)$ and global minimum at $(1, 18)$

b. $f'(x) = 12x^3 + 24x^2 - 12x - 24$
$= 12(x + 2)(x - 1)(x + 1)$
$f'(x) = 0 \Leftrightarrow x = -2, -1, 1$
$f'(x)$ is undefined $\Leftrightarrow x = -3, 2$.
$f''(x) = 36x^2 + 48x - 12 = 12(3x^2 + 4x - 1)$;
$f''(x) = 0 \Leftrightarrow x = -\dfrac{1}{3}(2 \pm \sqrt{7}) = 0.2152\ldots$ or $-1.5485\ldots$
$f''(x)$ is undefined $\Leftrightarrow x = -3, 2$.

c. $f''(-2) = 12[3(4) + 4(-2) - 1] = 36 > 0$, confirming local minimum.

d. Critical and inflection points occur only where f, f', or f'' is undefined (only at endpoints) or is zero (all such points are found above).

36. a. $f(x) = (x - 1)^5 + 4$, $x \in [-1, 3]$



Maximum $(3, 36)$, minimum $(-1, -28)$, plateau and points of inflection $(1, 4)$
Global maximum at $(3, 36)$ and global minimum at $(-1, -28)$

b. $f'(x) = 5(x - 1)^4$
$f'(x) = 0 \Leftrightarrow x = 1$; $f'(x)$ is undefined $\Rightarrow$ $x = -1, 3$.
$f''(x) = 20(x - 1)^3$;
$f''(x) = 0 \Leftrightarrow x = 1$; $f''(x)$ is undefined $\Rightarrow$ $x = -1, 3$.

c. $f''(1) = 20(1 - 1)^3 = 0$, so the test fails.

d. Critical and inflection points occur only where f, f', or f'' is undefined (only at endpoints) or is zero (all such points are found above).

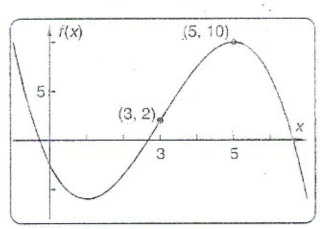
37. $f(x) = ax^3 + bx^2 + cx + d$: $f'(x) = 3ax^2 + 2bx + c$
$f''(x) = 6ax + 2b \Rightarrow f''(x) = 0$ at $x = -b/(3a)$
Because the equation for $f''(x)$ is a line with nonzero slope, $f''(x)$ changes sign at $x = -b/(3a)$, so there is a point of inflection at $x = -b/(3a)$.

38. $f(x)$ may not have a local maximum or minimum (if $f'(x)$ is never zero); if this is not the case, then the maximum and minimum occur where $f'(x) = 3ax^2 + 2bx + c = 0$, at

$$x = \frac{-2b \pm \sqrt{4b^2 - 4 \cdot 3a \cdot c}}{6a} = \frac{-b}{3a} \pm \frac{\sqrt{b^2 - 3ac}}{3a}$$

and the maximum and minimum occur at $\sqrt{b^2 - 3ac}/(3a)$ units on either side of the inflection point $-b/(3a)$ (see Problem 33).

39. $f(x) = ax^3 + bx^2 + cx + d$
$f'(x) = 3ax^2 + 2bx + c$; $f''(x) = 6ax + 2b$
Points of inflection at $(2, 3) \Rightarrow f''(3) = 0 \Rightarrow$
$18a + 2b = 0$
Maximum at $(5, 10) \Rightarrow f'(5) = 0 \Rightarrow 75a + 10b - c = 0$
$(3, 2)$ and $(5, 10)$ are on the graph $\Rightarrow$
$27a + 9b + 3c + d = 2$.
$125a + 25b + 5c + d = 10$
Solving this system of equations yields
$f(x) = -\dfrac{1}{2}x^3 + \dfrac{9}{2}x^2 - \dfrac{15}{2}x - \dfrac{5}{2}$.



The graph confirms maximum $(5, 10)$ and points of inflection $(3, 2)$.

40. $f(x) = ax^3 + bx^2 + cx + d$
$f'(x) = 3ax^2 + 2bx + c$; $f''(x) = 6ax + 2b$
Points of inflection at $(2, 7) \Rightarrow f''(2) = 0 \Rightarrow$
$12a + 2b = 0$
Maximum at $(-1, 61) \Rightarrow f'(-1) = 0 \Rightarrow$
$3a - 2b + c = 0$

28. a. $f(x) = 0.1x^4 - 3.2x + 7$
$f'(x) = 0.4x^3 - 3.2 = 0.4(x - 2)(x^2 + 2x + 4)$
$x^2 + 2x + 4$ has discriminant $= 2^2 - 4 \cdot 4 < 0$,
so $f'(x) = 0 \Leftrightarrow x = 2$ (critical point for $f(x)$).
$f''(x) = 1.2x^2$
$f''(x) = 0 \Leftrightarrow x = 0$ (critical point for $f'(x)$)

b. $f''(x)$ does not change sign at $x = 0$.
($f''(x) \geq 0$ for all x)

c. $f''(c) = 0$, but $f'(c) \neq 0$

29. a. $f(x) = xe^{-x}$
$f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1 - x)$
$f'(x) = 0 \Leftrightarrow x = 1$ (critical point for $f(x)$)
$f''(x) = xe^{-x} - 2e^{-x} = e^{-x}(x - 2)$
$f''(x) = 0 \Leftrightarrow x = 2$ (critical point for $f'(x)$)

b. Because $f(x)$ approaches its horizontal asymptote ($y = 0$) from above, the graph must be concave up for large x; but the graph is concave down near $x = 1$, and the graph is smooth; somewhere the concavity must change from down to up.

c. No. $e^{-x} \neq 0$ for all x, so $xe^{-x} = 0 \Leftrightarrow x = 0$.

30. a. $f(x) = x^2 \ln x$
$f'(x) = x + 2x \ln x = x(1 + 2 \ln x)$
$f(x)$ and $f'(x)$ are undefined at $x = 0$, so
$f'(x) = 0 \Leftrightarrow \ln x = -0.5 \Leftrightarrow x = e^{-0.5} =$
$0.6065\ldots$ (critical point for $f(x)$).
$f''(x) = 3 + 2 \ln x$
$f''(x) = 0 \Leftrightarrow \ln x = -1.5 \Leftrightarrow x = e^{-1.5} =$
$0.2231\ldots$ (critical point for $f'(x)$).

b. $\lim\limits_{x \to 0^+} x^2 \ln x = \lim\limits_{x \to 0^+} \dfrac{\ln x}{x^{-2}} = \lim\limits_{x \to 0^+} \dfrac{1/x}{-2x^{-3}}$
$= \lim\limits_{x \to 0^+} -0.5x^2 = 0$ by L'Hospital's rule.
$\lim\limits_{x \to 0^-} x^2 \ln x$ does not exist because $x^2 \ln x$ is undefined for $x < 0$.

c. All critical points from part a appear, although the inflection point at $x = e^{-1.5}$ is hard to see on the graph.

31. a. $f(x) = x^{5/3} + 5x^{2/3}$
$f'(x) = \dfrac{5}{3}x^{2/3} + \dfrac{10}{3}x^{-1/3} = \dfrac{5}{3}x^{-1/3}(x + 2)$
$f'(x) = 0 \Leftrightarrow x = -2$, and $f'(x)$ is undefined at $x = 0$ (critical points for $f(x)$).
$f''(x) = \dfrac{10}{9}x^{-1/3} - \dfrac{10}{9}x^{-4/3} = \dfrac{10}{9}x^{-4/3}(x - 1)$
$f''(x) = 0 \Leftrightarrow x = 1$ (critical point for $f'(x)$;
$f'(0)$ is undefined, so f' has no critical point at $x = 0$).

b. The y-axis ($x = 0$) is a tangent line because the slope approaches $-\infty$ from both sides.

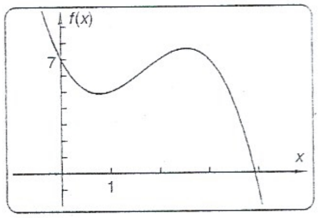
c. There is no inflection point at $x = 0$ because concavity is down for both sides, but there is an inflection point at $x = 1$.

32. a. $f(x) = x^{1.2} - 3x^{0.2}$
$f'(x) = 1.2x^{0.2} - 0.6x^{-0.8} = 0.6x^{-0.8}(2x - 1)$
$f'(x) = 0 \Leftrightarrow x = 0.5$, and $f'(x)$ is undefined at $x = 0$ (critical points for $f(x)$).
$f''(x) = 0.24x^{-0.8} + 0.48x^{-1.8} = 0.24x^{-1.8}(x + 2)$
$f''(x) = 0 \Leftrightarrow x = -2$ (critical point for $f'(x)$;
$f'(0)$ is undefined, so f' has no critical point at $x = 0$).

b. $f(0) = 0^{1.2} - 3 \cdot 0^{0.2} = 0$ has only one value.

c. Curved concave up because $f''(x) > 0$ for $x < -2$

33. a. $f(x) = -x^3 + 5x^2 - 6x + 7$



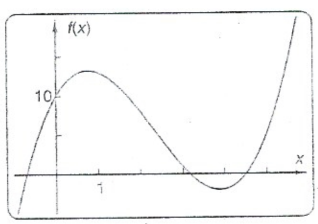
Maximum (2.5, 7.6), minimum (0.8, 4.9), points of inflection (1.7, 6.3)
No global maximum or minimum

b. $f'(x) = -3x^2 + 10x - 6$
$f'(x) = 0 \Leftrightarrow x = \dfrac{1}{3}(5 \pm \sqrt{7}) = 2.5485\ldots$ or $0.7847\ldots$
$f''(x) = -6x + 10$; $f''(x) = 0 \Leftrightarrow x = \dfrac{5}{3} =$
$1.666\ldots$

c. $f''(0.7847\ldots) = -6(0.7847\ldots) + 10 =$
$5.2915\ldots > 0$, confirming local minimum.

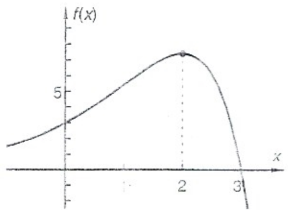
d. Critical and inflection points occur only where f, f', or f'' is undefined (no such points exist) or is zero (all such points are found above).

34. a. $f(x) = x^3 - 7x^2 + 9x + 10$



Maximum (0.8, 13.2), minimum (3.9, −2.1), points of inflection (2.3, 5.6)
No global maximum or minimum

21. $f(x) = 3e^x - xe^x$
$f'(x) = 3e^x - e^x - xe^x = e^x(2 - x)$
$f'(2) = e^2(2 - 2) = 0 \Rightarrow$ critical point at $x = 2$
$f''(x) = 2e^x - e^x - xe^x = e^x(1 - x)$
$f''(2) = e^2(1 - 2) = -7.3890... < 0$
$\therefore$ local maximum at $x = 2$



The graph confirms a maximum at $x = 2$.

22. $f(x) = -\sin\dfrac{\pi}{4}x$

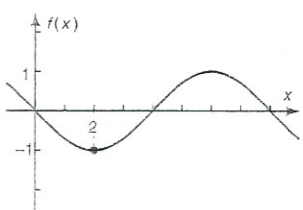
$f'(x) = -\dfrac{\pi}{4}\cos\dfrac{\pi}{4}x$

$f'(2) = -\dfrac{\pi}{4}\cos\dfrac{\pi}{4}(2) = 0 \Rightarrow$ critical point at $x = 2$

$f''(x) = \dfrac{\pi^2}{16}\sin\dfrac{\pi}{4}x$

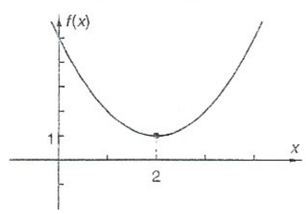
$f''(2) = \dfrac{\pi^2}{16}\sin\dfrac{\pi}{4}(2) = 0.6168... > 0$

$\therefore$ local minimum at $x = 2$



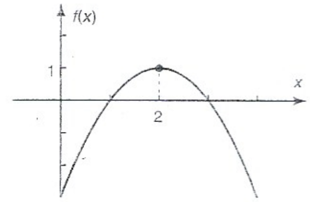
The graph confirms a minimum at $x = 2$.

23. $f(x) = (2 - x)^2 + 1$
$f'(x) = -2(2 - x)$
$f'(2) = -2(2 - 2) = 0 \Rightarrow$ critical point at $x = 2$
$f''(x) = 2 \Rightarrow f''(2) = 2 > 0$
$\therefore$ local minimum at $x = 2$



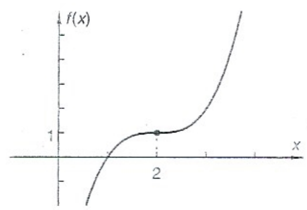
The graph confirms a minimum at $x = 2$.

24. $f(x) = -(x - 2)^2 + 1$
$f'(x) = -2(x - 2)$
$f'(2) = -2(2 - 2) = 0 \Rightarrow$ critical point at $x = 2$
$f''(x) = -2 \Rightarrow f''(2) = -2 < 0$
$\therefore$ local maximum at $x = 2$



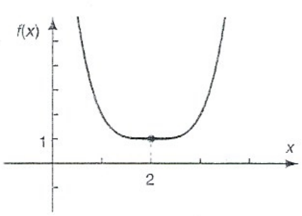
The graph confirms a maximum at $x = 2$.

25. $f(x) = (x - 2)^3 + 1$
$f'(x) = 3(x - 2)^2$
$f'(2) = 3(2 - 2)^2 = 0 \Rightarrow$ critical point at $x = 2$
$f''(x) = 6(x - 2)$
$f''(2) = 6(2 - 2) = 0$, so the test fails.
$f'(x)$ goes from positive to positive as x increases through 2, so there is a plateau at $x = 2$.



The graph confirms a plateau at $x = 2$.

26. $f(x) = (2 - x)^4 + 1$
$f'(x) = -4(2 - x)^3$
$f'(2) = -4(2 - 2)^3 = 0 \Rightarrow$ critical point at $x = 2$
$f''(x) = 12(2 - x)^2$
$f''(2) = 12(2 - 2)^2 = 0$, so the test fails.
$f'(x)$ changes from negative to positive as x increases through 2, so there is a local minimum at $x = 2$.



The graph confirms a minimum at $x = 2$.

27. a. $f(x) = 6x^5 - 10x^3$
$f'(x) = 30x^4 - 30x^2 = 30x^2(x + 1)(x - 1)$
$f'(x) = 0 \Leftrightarrow x = -1, 0,$ or 1 (critical points for $f(x)$)
$f''(x) = 120x^3 - 60x = 60x(\sqrt{2}x + 1)(\sqrt{2}x - 1)$
$f''(x) = 0 \Leftrightarrow x = 0, \pm\sqrt{1/2}$ (critical points for $f'(x)$)

b. The graph begins after the f-critical point at $x = -1$; the f'-critical point at $x = -\sqrt{1/2}$ is shown, but is hard to see.

c. $f'(x)$ is negative for both $x < 0$ and $x > 0$.

[redacted]

More Max/Min Problems

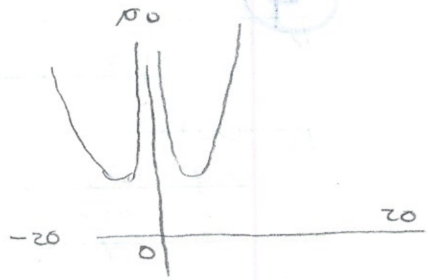
① $xy = -12$

$y = \frac{-12}{x}$

$S = x^2 + y^2$

$S(x) = x^2 + \left(\frac{-12}{x}\right)^2$

$= x^2 + 144x^{-2}$



$S'(x) = 2x - 288x^{-3}$

$2x^{-3}(x^4 - 144) = 0$

$2x^{-3} = 0$ UGH.

$(x^2 - 12)(x^2 + 12) = 0$

$x = \pm\sqrt{12}$ ± 3.48

$y = \pm\sqrt{12}$

$x = -3.46;\ 3.46$

$S(x) = 24$

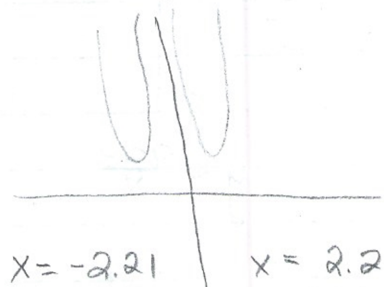
② $x = 2y^2$

$d^2 = \Delta x^2 + \Delta y^2$

$= (x-10)^2 + (y-0)^2$

$d^2(x) = (2y^2 - 10)^2 + y^2$

$= 4y^4 - 40y^2 + 100 + y^2$



$d^2(x) = 4y^4 - 39y^2 + 100$

$d'(x) = 16y^3 - 78y$

$2y(8y^2 - 39)$

$2y = 0$

$y = 0$

$8y^2 = 39$

$y^2 = \frac{39}{8}$

$y = \pm 2.21$

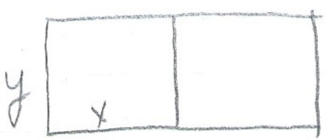
$x = -2.21$ $y = 4.94$

$x = 2.21$ $y = 4.94$

Points are

$(9.76, 2.21)\ (9.76, -2.21)$

③



$A = 900\,ft^2$

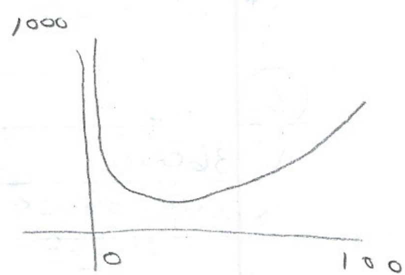
$900 = xy$

$y = \frac{900}{x}$

$P = 4x + 3y$

$P(x) = 4x + 3\left(\frac{900}{x}\right)$

$P(x) = 4x + 2700x^{-1}$



$P'(x) = 4 - 2700x^{-2}$

$\frac{4}{1} = \frac{2700}{x^2}$

$4x^2 = 2700$

$x^2 = 675$

$\boxed{\begin{aligned} x &= 25.98\ ft \\ y &= 34.64\ ft \end{aligned}}$

$x = 25.98$

$f(x) = 20.7.85$

[redacted]

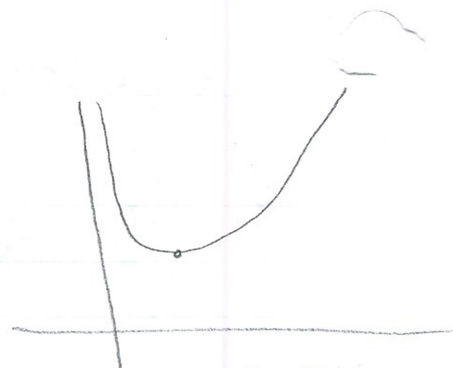
4

$$C = 2(4x) + 1(By) \quad +2(2y)$$

$$C = 8x + 5y$$

$$C(x) = 8x + 5\left(\frac{900}{x}\right)$$

$$= 8x + 4500x^{-1}$$



$x = 23.7$

$C(x) = \$379.47$

$A = 900\,ft^2$

$x \cdot y = 900$

$y = \frac{900}{x}$

$$C'(x) = 8 - 4500x^{-2}$$

$$8 = 4500x^{2}$$

$$x^2 = \frac{4500}{8}$$

$$\boxed{\begin{array}{l} x = 23.7 \\ y = 37.9 \end{array}}$$

5.

$y = \frac{900}{x}$

$$P = 6x + 4y$$

$$P(x) = 6x + 4\left(\frac{900}{x}\right)$$

$$= 6x + 3600x^{-1}$$

$x = 24.49$

$P(x) = 293.94$

$$P'(x) = 6 - 3600x^{-2}$$

$$\frac{6}{1} = \frac{3600}{x^2}$$

$$6x^2 = 3600$$

$$x^2 = 600$$

$$\boxed{\begin{array}{l} x = 24.49\text{ ft} \\ y = 36.74\text{ ft.} \end{array}}$$

6

$V = 36000$ oops

$2x \cdot x \cdot y = 36000$

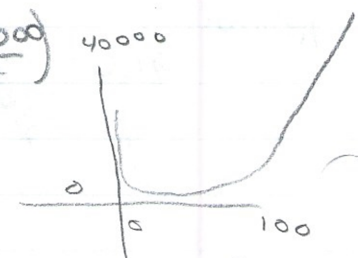
$y = \frac{18000}{x^2}$

$$SA = 2x^2 + 2xy + 4xy$$

$$SA(x) = 2x^2 + 2x\left(\frac{18000}{x^2}\right) + 4x\left(\frac{18000}{x^2}\right)$$

$$= 2x^2 + 36000x^{-1} + 72000x^{-1}$$

$$= 2x^2 + 108000x^{-1}$$



$x = 30$

$SA(x) = 5400$

$$SA'(x) = 4x - 108000x^{-2}$$

$$4x^{-2}(x^3 - 27000)$$

$8x^{-2} = 0$ NO

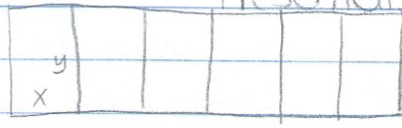
$$x^3 = 2700$$

$$\boxed{x = 30 \qquad 2x = 60 \qquad y = 20}$$

Rebecca Russell
M24

#6

Presentation Problem (#2)

2) 

$A = 350\,ft^2$

a) $P = 12x + 7y$

\# of horizontal walls — \# of vertical walls

$A: \quad 350 = xy$ Insert known area into area equation

$\frac{350}{x} = y$ Put it into terms of x

$L(x) = 12x + 7\left(\frac{350}{x}\right)$ Insert the equation you found for y ↑

$= 12x + 2450x^{-1}$ simplify

Algebraically:

$P(x) = 12x + 2450x^{-1}$ ← Perimeter Equation

$P'(x) = 12 - 2450x^{-2}$ Differentiate

$0 = 12 - \frac{2450}{x^2}$

$-12 = -\frac{2450}{x^2}$

$12 = \frac{2450}{x^2}$

$\frac{12x^2}{12} = \frac{2450}{12}$

Simplify the equation

$\boxed{x = 14.2887}$ Solve for x

$y = \frac{350}{x}$ ← Area equation

$y = \frac{350}{14.2887}$ Replace value for x

$\boxed{y = 24.4949}$ Solve for y

$P''(x) = \frac{4900}{x^3}$

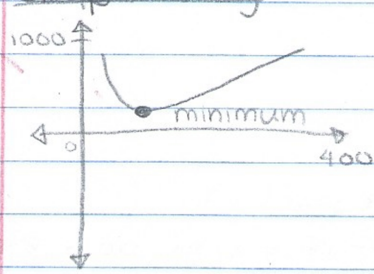
$P''(14.2887) = +$

concave up

minimum

Since $P''(14.2887) > 0$, the curve is concave up which means $x = 14.2887$ is a minimum

Graphically



x-value ↓ perimeter ↓

$(14.2887, 342.9286)$

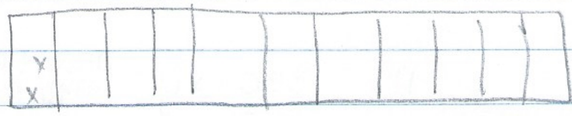
$y = \frac{350}{x} = \frac{350}{14.2887} = \boxed{24.4949}$

The dimensions of the hotel are $14.3' \times 24.5'$

b) on the back

I want to make a copy!

I. Ten Rooms



$A = 350 ft^2$

Area equation

$\frac{350}{x} = \frac{xy}{\cancel{x}}$

$P = 20x + 11y$ ← Perimeter equation

$\uparrow$ # of horizontal walls $\uparrow$ # of vertical walls

$= 20x + 11\left(\frac{350}{x}\right)$ ← substitute area equation for y

$= 20x + 3850x^{-1}$ ← simplify

$P'(x) = 20 - 3850x^{-2}$ ← differentiate

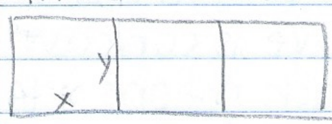
$= \sqrt{\frac{3850}{20}}$ ← simplify and solve for x (see part a)

$y = \frac{350}{x} = \frac{350}{13.8744}$ ← plug x into area equation

$\boxed{x = 13.8744 \text{ ft}}$ $\boxed{y = 25.2262 \text{ ft}}$

The dimensions of the hotel are 13.9' x 25.2'

II Three Rooms



$A = 350 ft^2$

Area equation

$\frac{350}{x} = \frac{\cancel{x}y}{\cancel{x}}$

$P = 6x + 4y$ ← Perimeter equation

$\uparrow$ # of horizontal walls ↖ # of vertical walls

$= 6x + 4\left(\frac{350}{x}\right)$ ← substitute area equation for y

$= 6x + 1400x^{-1}$ ← simplify

$P'(x) = 6 - 1400x^{-2}$ ← differentiate

$= \sqrt{\frac{1400}{6}}$ ← simplify + solve for x (see part a)

$y = \frac{350}{x} = \frac{350}{15.2753}$ ← plug x into area equation

$\boxed{x = 15.2753 \text{ ft}}$ $\boxed{y = 22.9129 \text{ ft}}$

The dimensions of the hotel are 15.3' x 22.9'