CALCULUS FINAL EXAM REVIEW Ch. 6

① indeterminate form and quotient

② ⓐ $M'(t) = 1000(1.06)^t \cdot \ln 1.06$

ⓑ

t	$M'(t)$
0	58.2689...
10	104.3507...
100	19,770.7619....

\$/yr.

ⓒ

t	$M(t)$
0	1000
10	1,790.847...
100	339,302.0835....

Rate of increase is increasing as the amount increases

③ ⓐ $\lim_{x\to\infty} \frac{5x^2-11x+7}{4+3x-2x^2} \quad \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{10x-11}{3-4x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{10}{-4} = -5/2$

ⓑ $\lim_{x\to\infty} \frac{7x^2-4}{3-4x^4} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{14x}{-16x^3} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{14}{-48x^2} = \frac{14}{\infty} = 0$

ⓒ $\lim_{x\to 0} \frac{1-\cos x}{x} = \frac{1-1}{0} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\sin x}{1} = \frac{\sin 0}{1} = 0$

④ $\ln x = \int_1^x \frac{1}{t}\,dt$

⑤ ⓐ $\int 5^x dx = \frac{1}{\ln 5} \cdot 5^x + C$

ⓑ $\int e^{3x} dx = \frac{1}{3} e^{3x} + C$

ⓒ $\frac{1}{2}\int 2x e^{x^2} \cdot dx \qquad u = x^2, \quad du = 2x\,dx$

$\frac{1}{2}\int e^u du = \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2} + C$

ⓓ $\int \frac{(\ln x)^5}{x} dx \qquad u = \ln x, \quad du = \frac{1}{x} dx$

$\int u^5 \cdot du = \frac{1}{6} u^6 + C = \frac{1}{6}(\ln x)^6 + C$

ⓔ $\int_1^7 \frac{1}{p} dp = \ln|p| + c \Big|_1^7$

$= \ln 7 - \ln 1 = 1.9459$

ⓕ $\int \tan 3x \cdot dx = \frac{1}{3} \ln|\sec 3x| + C$

⑥ ⓐ $f(x) = \log_{10}(\tan x)$

$f(x) = \frac{\ln(\tan x)}{\ln 10}$

$f'(x) = \frac{1}{\ln 10} \cdot \frac{1}{\tan x} \cdot \sec^2 x$

$f'(x) = \frac{\sec^2 x}{\ln 10 \cdot \tan x}$

ⓑ $y = e^{5x}$

$y' = 5e^{5x}$

ⓒ $f(x) = 10^{\sin x}$

$y = 10^{\sin x}$

$\ln y = \sin x \cdot \ln 10$

$\frac{1}{y} \cdot y' = \ln 10 (\cos x)$

$y' = 10^{\sin x}(\ln 10 \cdot \cos x)$

ⓓ $f(x) = x^3 \cdot \ln x$

$f'(x) = 3x^2 \cdot \ln x + x^3 \cdot \frac{1}{x}$

$= 3x^2 \ln x + x^2$

or $= x^2(3\ln x + 1)$

ⓔ $y = 5e^{\ln x^3}$

$y = 5x^3$

$y' = 15x^2$

ⓕ $y = \ln(\sin^5 x)$

$y' = \frac{1}{\sin^5 x} \cdot 5\sin^4 x \cdot \cos x$

$y' = \frac{5\cos x}{\sin x} = 5\cot x$

ⓖ $y = x^x$

$\ln y = x \ln x$

$\frac{1}{y} \cdot y' = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$y' = x^x(\ln x + 1)$

ⓗ $y = e^{5\ln x}$

$y = e^{\ln x^5}$

$y = x^5$

$y' = 5x^4$

ⓘ $y = \ln(\csc x)$

$y' = \frac{1}{\csc x} \cdot -\csc x \cdot \cot x$

$y' = -\cot x$

REVIEW Ch. 6 continued

7 (a) $y = (5x-7)^3(3x+1)^5$

$\ln y = 3\ln(5x-7) + 5\ln(3x+1)$

$\frac{1}{y}\cdot y' = 3\cdot\frac{1}{5x-7}\cdot 5 + 5\cdot\frac{1}{3x+1}\cdot 3$

$y' = (5x-7)^3(3x+1)^5\left[\frac{15}{5x-7} + \frac{15}{3x+1}\right]$

$= 15(5x-7)^3(3x+1)^5\left[\frac{1}{5x-7} + \frac{1}{3x+1}\right]$

(b) $y = \frac{(x^2-3)^3}{(4x^5+5x)^7}$

$\ln y = 3\ln(x^2-3) - 7\ln(4x^5+5x)$

$\frac{1}{y}\cdot y' = 3\frac{1}{x^2-3}\cdot 2x - 7\cdot\frac{1}{4x^5+5x}(20x^4+5)$

$y' = \frac{(x^2-3)^3}{(4x^5+5x)^7}\left[\frac{6x}{x^2-3} - \frac{7(20x^4+5)}{4x^5+5x}\right]$

(c) $y = x^{\ln x}$

$\ln y = \ln x\cdot\ln x$

$\ln y = (\ln x)^2$

$\frac{1}{y}\cdot y' = 2\ln x\cdot\frac{1}{x}$

$y' = x^{\ln x}\left(\frac{2\ln x}{x}\right)$

(d) $y = (3x-4)^{\cos x}$

$\ln y = \cos x\cdot\ln(3x-4)$

$\frac{1}{y}\cdot y' = -\sin x\cdot\ln(3x-4) + \cos x\cdot\frac{1}{3x-4}\cdot 3$

$y' = (3x-4)^{\cos x}\left[\frac{3\cos x}{3x-4} - \sin x\cdot\ln(3x-4)\right]$

CALCULUS FINAL EXAM REVIEW Ch. 7

(1) (a) $\frac{dy}{dx} = 2x - 1 \quad (2,-1)$

$\int dy = \int (2x-1)\,dx$

$y = x^2 - x + C$

$-1 = 2^2 - 2 + C$

$-3 = C$

$\boxed{y = x^2 - x - 3}$

(b) $\frac{dy}{dx} = \frac{6x - x^3}{2y} \quad (0,3)$

$\int 2y \cdot dy = \int (6x - x^3)\,dx$

$y^2 = 3x^2 - \frac{1}{4}x^4 + C$

$3^2 = 0 - 0 + C$

$9 = C$

$y = \sqrt{3x^2 - \frac{1}{4}x^4 + 9}$

(c) $\frac{dy}{dt} = t^2 y^4 \quad (1,1)$

$\int y^{-4}\,dy = \int t^2\,dt$

$-\frac{1}{3}y^{-3} = \frac{1}{3}t^3 + C$

$-\frac{1}{3} = \frac{1}{3} + C$

$-\frac{2}{3} = C$

$-\frac{1}{3} \cdot y^{-3} = \frac{1}{3}t^3 - \frac{2}{3}$

$3 \cdot \frac{-y^{-3}}{3} = \frac{t^3 - 2}{3} \cdot 3$

$-y^{-3} = t^3 - 2$

$y^{-3} = 2 - t^3$

$y = (2 - t^3)^{-1/3}$

(2) (a) $\frac{dB}{dt} = k \cdot B$

$\int \frac{1}{B}\,dB = \int k\,dt$

$e^{\ln|B|} = e^{kt + C}$

$|B| = e^{kt} \cdot e^{C}$

$B = Ce^{kt}$

(b) $(0, 10)$ in thousands

$10 = Ce^{k \cdot 0}$

$10 = C$

$B = 10e^{kt}$

$42 = 10e^{k \cdot 10} \quad (10, 42)$

$4.2 = e^{10k}$

$\ln 4.2 = 10k$

$k = \frac{\ln 4.2}{10} = 0.1435$

$B = 10e^{0.1435t}$

(c) $t = 25$

$B(25) = 361{,}436$

time to double?

$20 = 10e^{0.1435t}$

$2 = e^{0.1435t}$

$\ln 2 = 0.1435 \cdot t$

$t = \frac{\ln 2}{0.1435} = 4.8303$ days.

(3) function

(4) E (5) A

(6) (a) $\frac{dF}{dt} = k \cdot F$

$F = C \cdot e^{k \cdot t}$

$\boxed{\text{See 2a}}$

(b) $(0, 130)$ $\boxed{F = 130e^{-0.0351t}}$

$C = 130$

$F = 130e^{k \cdot t}$

$(3, 117)$

$117 = 130 \cdot e^{k \cdot 3}$

$\frac{117}{130} = e^{3 \cdot k}$

$\ln\left(\frac{117}{130}\right) = 3 \cdot k$

$k = -0.0351$

(c) F after 15 min

$F(15) = 76.7869^{\circ}$

(d) $68 = 130e^{-0.0351t}$

$68/130 = e^{-0.0351t}$

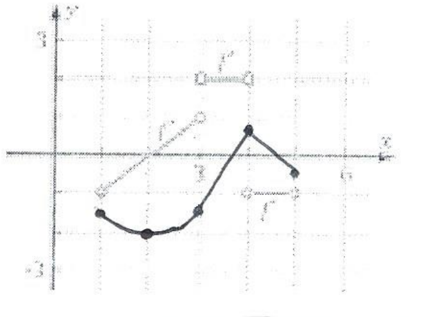
$\frac{\ln(68/130)}{-0.0351} = t$

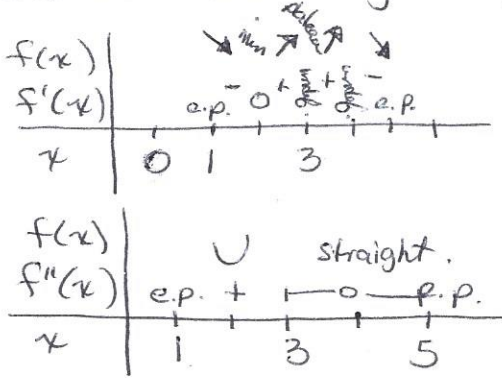
$t = 18.462$ min.

CALCULUS FINAL EXAM REVIEW Ch. 8

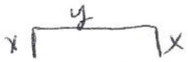
(1) C (2) A (3) The second derivative changes sign around @ $x=c$ and $f''(c)=0$

(4)





(5.)



$2x+y=P=800$

$A=x\cdot y$

$2x+y=800$

$y=800-2x$

$y=800-2(200)$

$=400\text{m}$

$A=x\cdot y$

$A(x)=x(800-2x)$

$=800x-2x^2$

$A'(x)=800-4x$

$0=800-4x$

$4x=800$

$x=200\text{m}$

$A=400\cdot 200=80{,}000\,\text{m}^2$

$200\text{m}\times 400\text{m}$

Why a max?

$A''(x)=-4$

If $\frac{d^2y}{dx^2}$ is negative, then curve is concave down at all points.

$\therefore$ $x=200$ is a maximum

(6). $V=\pi r^2 h$

$SA=2\pi r^2+2\pi rh$

$SA(r)=2\pi r^2+2\pi r\left(\frac{1500}{\pi r^2}\right)$

$=2\pi r^2+3000r^{-1}$

$SA'(r)=4\pi r-3000r^{-2}$

$0=4\pi r-\frac{3000}{r^2}$

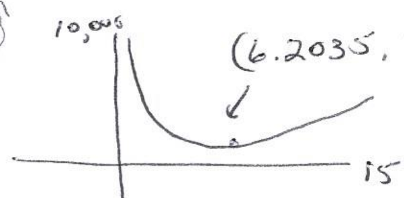
$\frac{3000}{r^2}=4\pi r$

$3000=4\pi r^3$

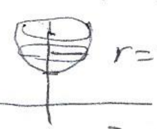
$r=\sqrt[3]{\frac{3000}{4\pi}}=6.2035$

$1500=\pi r^2 h$

$h=\frac{1500}{\pi r^2}=12.4070$



(7)



$r=x$

$y=x^2+2$

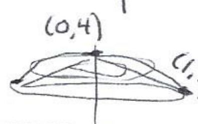
$y=5$

$x=\sqrt{y-2}$

$dV=\pi r^2\cdot dy$

$V=\pi\int_2^5(\sqrt{y-2})^2\,dy=4.5\pi=14.1372\,\text{un}^3$

(8)



r_O = curve

r_I = line

$y=4-x \rightarrow x=4-y$

$y=4-x^2 \rightarrow x=\sqrt{4-y}$

$dV=\pi(R_o^2-r_I^2)\,dy$

$V=\pi\int_3^4\left[(\sqrt{4-y})^2-(4-y)^2\right]dy=\frac{1}{6}\pi=0.5236\,\text{un}^3$

(9)

$s=x^{\frac{1}{5}}-x^2$

$x^{\frac{1}{5}}=x^2$

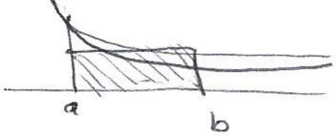
@ $x=0$ and $x=1$

$dV=s^2\cdot dx$

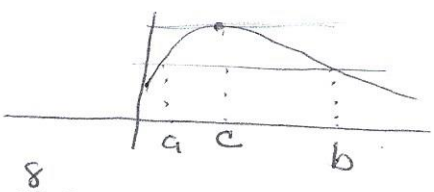
$V=\int_0^1\left[x^{1/5}-x^2\right]^2dx=0.2893\,\text{un}^3$

CALCULUS FINAL EXAM REVIEW Ch. 10

(1) Mean Value Th for Integrals: If $f(x)$ is continuous on the closed interval $[a,b]$ then there is at least one point $x = c$ for which $f(c) = \frac{\int_a^b f(x)\,dx}{b-a}$ or
$f(c)(a-b) = \int_a^b f(x)\,dx$. [area under curve = area of rectangle formed ~~with~~ with average value]



Mean Value Th for Derivatives: If $f(x)$ is continuous in the closed interval $[a,b]$ and differentiable in the open interval (a,b), then there is at least one point $x = c$ in (a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$
[Slope of secant line thru $(a, f(a))$ and $(b, f(b))$ = slope of tangent line at $x = c$



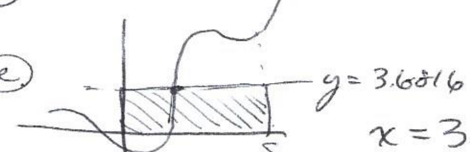
(2) (a) $\int_0^8 [x + 2\sin(x-3)]\,dx = 29.4527$

(b) $\int_0^8 |x + 2\sin(x-3)|\,dx = 31.6244$

(c) $\frac{\int_0^8 (\quad)\,dx}{8-0} = 3.6816$

(d) $3.6816 = x + 2\sin(x-3)$

(e)



$x = 3.2285$

area under curve = area of rectangle

$\int_0^8 29.4527 = 29.4527$
8×3.6816

(3) $T(x) = \frac{1}{20}(16+x^2)^{1/2} + \frac{1}{55}(9-x)$

$T'(x) = \frac{1}{40}(16+x^2)^{-1/2} \cdot 2x + \left(-\frac{1}{55}\right) = 0$

$\frac{x}{20\sqrt{16+x^2}} = \frac{1}{55}$

$55x = 20\sqrt{16+x^2}$

$\left(\frac{55x}{20}\right)^2 = \left(\sqrt{16+x^2}\right)^2$

$\frac{121}{16}x^2 = 16 + x^2$

$\frac{105}{16}x^2 = 16$

$x^2 = \frac{256}{105} = 1.5614$

Claude should join the dirt road to the highway 1.5614 miles from where a road perpendicular to the highway would intersect.

(4) (a) distance $\int_0^3 |v(t)|\,dt = 30.0564$
displacement $\int_0^3 v(t)\,dt = 25.6294$

(b) $v'(t) = a(t) = +10(2^{-t})\ln 2$
$a(0) = 6.9315$ ft/sec^2

(c) $v(0) = -$ $a(0) = +$
slowing down - signs are different

(5) $C(x) = 300\sqrt{144+x^2} + 175(50-x)$

$C(x) = 300(144+x^2)^{1/2} + 8750 - 175x$

$C'(x) = 150(144+x^2)^{-1/2} \cdot 2x - 175$

$0 = \frac{300x}{\sqrt{144+x^2}} - 175$

$175 = \frac{300x}{\sqrt{144+x^2}}$

$\left(\frac{175\sqrt{144+x^2}}{175}\right)^2 = \left(\frac{300x}{175}\right)^2$

$144 + x^2 = \frac{144}{49}x^2$

$144 = \frac{95}{49}x^2$

$x = \sqrt{\frac{144}{95/49}} = 8.6182$