

Review of the Derivative Rules

Find each derivative function. Apply the product, quotient, chain, and trigonometric rules as appropriate.

1. $y = (3x^2 + 5x - 3)^7$

$$y' = \frac{(42x+35) \cdot (3x^2+5x-3)^6}{(3x^2+5x-3)}$$

2. $y = (2x^2 + 5x)(3x^2 + 7)$

$$y' = 24x^3 + 45x^2 + 28x + 35$$

3. $y = \frac{3x+4}{2x-5}$

$$y' = \frac{-23}{(2x-5)^2}$$

4. $y = \frac{4}{x^2} - \frac{6}{x} + 8$

$$y' = -\frac{8}{x^3} + \frac{6}{x^2}$$

5. $y = \left(\frac{2x}{3x-7}\right)^4$

$$y' = -56 \left(\frac{2x^3}{(3x-7)^5}\right)$$

6. $y = \sin(5x - 3)$

$$y' = 5\cos(5x-3)$$

7. $f(x) = \sec(3x^2 + 4)$

$$y' = \frac{6x}{\sec(3x^2+4)} \tan(3x^2+4)$$

8. $f(x) = \tan^3(6x^2)$

$$y' = \frac{36x \tan^2(6x^2)}{\sec^2(6x^2)}$$

9. $f(x) = \cos^4(3x^5)$

$$y' = -60x^4 \cos^3(3x^5) \cdot \sin(3x^5)$$

10. $f(x) = 2x^3 \cos(17x)$

$$f'(x) = 6x^2 \cos(17x) - 34x^3 \sin(17x)$$

11. $f(x) = \frac{\csc(3x)}{x^2}$

$$f'(x) = \frac{-3x^2 \csc(3x) \cot(3x) - 2x \csc(3x)}{x^4}$$

12. $f(x) = (x^3 - 9)^4 (x^2 + 4)^5$

$$y' = 12x^2 (x^3-9)^3 (x^2+4)^5 + 10x(x^3-9)^4 (x^2+4)^4$$

13. Write the equation of the line tangent to the curve $y = 3x^3 - 2x + 4$ at $(2, 24)$

$$y' = 9x^2 - 2$$

$$y'(2) = 34$$

$$y = 34(x-2) + 24$$

14. Write the equation of the line tangent to the curve $y = \sqrt{3x+4}$ at $(7, 5)$

$$y' = \frac{1}{2}(3x+4)^{-\frac{1}{2}} \cdot 3$$

$$y'(7) = \frac{3}{10}$$

$$y = \frac{3}{10}(x-7) + 5$$

15. An object thrown vertically up with an initial velocity of 64 ft/sec from a building 96 feet high satisfies the position function $y = -16t^2 + 64t + 96$.

a) Explain each coefficient.

$$\begin{array}{ccc} -16 & 64 & 96 \\ T & \downarrow & \downarrow \\ -\frac{1}{2}g^2 & v_0 & h_0 \end{array}$$

b) When did the object reach its maximum height? Use calculus as well as "old" methods.

$$\begin{aligned} y' &= -32t + 64 = 0 \\ t &= 2 \text{ sec} \end{aligned}$$

c) What was the maximum height?

$$y(2) = 160 \text{ ft}$$

d) When did the object hit the ground?

$$5.16 \text{ sec}$$

e) What was the velocity of the object when it hit the ground? $y'(5.16) = -101.19$

$$16. y = (2-9x)^{15}$$

$$17. f(x) = (4x+7)^{23}$$

$$18. y = \frac{2x+3}{x^2-5x}$$

$$y' = -135(2-9x)^{14}$$

$$f'(x) = 92(4x+7)^{22}$$

$$y' = \frac{-2x^2-6x+15}{(x^2-5x)^2}$$

$$19. y = \sin^3 x$$

$$20. f(x) = \cos^5 x$$

$$21. f(x) = \sin(3x^2 + 11)$$

$$y' = 3 \sin^2 x \cos x$$

$$f'(x) = -5 \cos^4 x \sin x$$

$$f'(x) = 6x \cos(3x^2 + 11)$$

$$22. f(x) = \sin^3(\cos x)$$

$$23. y = \frac{\cos x}{3x^2+1}$$

$$24. y = \cos^3(5x^2)$$

$$f'(x) = -3 \sin^2(\cos x) \cdot \cos(\cos x) \cdot \sin x$$

$$y' = -30x \cos^2(5x^2) \sin(5x^2)$$

$$y' = \frac{-\sin x(3x^2+1) - 6x \cos x}{(3x^2+1)^2}$$

$$25. y = 314$$

$$y' = 0$$