# Chapter 6

## Counting, Probability, and Inference

### Chapter Review (pp. 432–435)

1. There are 21 possible outcomes where $d \geq 0$ and 15 possible outcomes where $d < 0$, so $P(d < 0) = \frac{15}{36} = \frac{5}{12}$.

2. Since all of the numbers on the die are integers, all of the outcomes are integers. Therefore, $P(d \text{ is an integer}) = 1$.

3. There are 6 possible outcomes for rolling doubles, so $P(\text{doubles}) = \frac{6}{36} = \frac{1}{6}$.

4. A sum of 2 is prime, and the odd numbers 3–11 are prime other than 9. Since there are 4 possible outcomes with a sum of 9, $P(\text{sum is prime}) = \frac{19}{36} - \frac{4}{36} = \frac{15}{36} = \frac{5}{12}$.

5. $P(\text{HHH}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

6. {HHT, HTH, THH}; $P(\text{2 heads}) = \frac{3}{2^3} = \frac{3}{8}$

7. $P(\text{at least 1 tail}) = 1 - P(\text{HHH}) = 1 - \frac{1}{2^3} = 1 - \frac{1}{8} = \frac{7}{8}$

8. $np = 1200 \cdot 0.013 \approx 16$

9. $np = 230 \cdot \frac{1}{6} = 38.\overline{3}$

10. $np = 532 \cdot 0.047 \approx 25$

11. $8! = 40{,}320$

12. ${}_7P_3 = 210$

13. ${}_7P_5 = 2520$

14. ${}_3P_1 \cdot {}_5P_1 \cdot {}_2P_1 = 30$

15. There are 2 possible outcomes for every toss, so for 8 tosses, $2^8 = 256$.

16. $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

17. $\frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 3 \cdot 2} = 21$

18. $\frac{100 \cdot 99 \cdot 98}{50 \cdot 49 \cdot 11 \cdot 10 \cdot 9} = \frac{2}{5}$

19. $\frac{54 \cdot 53 \cdot 52 \cdot 51 \cdot 50!}{50!} = 54 \cdot 53 \cdot 52 \cdot 51$

20. $\frac{(a+2)(a+1)!}{(a+1)!} = a + 2$

21. $\frac{12!}{(12-3)!} = \frac{12 \cdot 11 \cdot 10 \cdot 9!}{9!} = 12 \cdot 11 \cdot 10 = 1320$

22. $\frac{7!}{(7-5)!} = \frac{7!}{2} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$

23. $\frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$

24. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.6} = 0.08\overline{3}$

25. If the events are complementary, then they are mutually exclusive and $P(A \cap B) = 0$.

26. If $A$ and $B$ were mutually exclusive, then $P(A) + P(B) = P(A \cup B) \leq 1$, but here $P(A) + P(B) = 1.3$, so $A$ and $B$ cannot be mutually exclusive.

27. $P(\text{not } (A \text{ or } B)) = 1 - 0.25 - 0.3 = 0.45$

28. The possible outcomes are: {1, 4, 8, 9, 16, 25, 27, 36, 49, 64, 81, 100}, so $P(\text{perfect square or perfect cube}) = \frac{12}{100} = 0.12$.

29. $P(A \cap B) = P(A) \cdot P(B)$, so the events are independent.

30. The events are mutually exclusive, so they are dependent.

31. $P(A) = \frac{1}{6}$, $P(B) = \frac{6}{36}$, and $P(A \cap B) = \frac{1}{36}$; $P(A \cap B) = P(A) \cdot P(B)$, so the events are independent.

32. a. Selections with replacement are independent events because each selection is not affected by what has been selected previously, so the events are independent.

    b. Because the first selected sock was not replaced, the selection of the second sock is dependent on the selection of the first. So the events are dependent.

33. The announcer is incorrectly applying the "law of averages," assuming that the observed count should approach the expected count.

34. The relative frequency of 2 is $\frac{1699}{17{,}280}$. Over the long run, the Law of Large Numbers states that the relative frequency of 2 will approach $\frac{1}{10}$.

**35.** expected winnings:
$(0)(0.999) + (700)(0.001) = \$0.70$;
Someone who plays every day has expected winnings of $(\$0.70)(365) = \$255.5$ in one year, but spends \$365 during the year. In the long run, the winnings will approach \$0.70 per lottery ticket and therefore yield a profit of \$0.30 per lottery ticket for the state.

**36.** **a.** {T1, T2, T3, T4, T5, T6, H1, H2, H3, H4, H5, H6}

**b.** H1, H2, H3, H4, H5, H6, T4, T6

**c.** H1, H2, T1, T2; so, 4 outcomes

**37.** Answers vary. Sample: The integers from 0 to 359, since the sum of the central angles of a circle is 360 degrees.

**38.** The entire set of TV programs available at that time.

**39.** All 366 days of the year (do not forget February 29th).

**40.** maple: $\frac{16}{102} \approx 0.157$, so about 15.7%, oak: $\frac{8}{68} \approx 0.118$, so about 11.8%, ash: $\frac{17}{85} = 0.2$, so 20%; Oak seems to be the sturdiest species.

**41.** $\frac{68}{102 + 68 + 85} \approx 0.267$, so about 26.7%;
$\frac{60}{86 + 60 + 68} \approx 0.280$, so about 28.0%

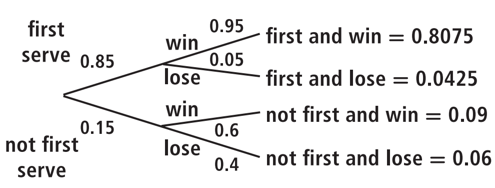
**42.** $\frac{16 + 8 + 17}{102 + 68 + 85} \approx 0.161$, so about 16.1%

**43.** $\frac{15}{1024} \approx 0.015$

**44.** $(0.7)^4 = 0.2401$

**45.** $0.08 \cdot 0.120 = 0.0096$, so 0.96%

**46.** **a.**




**b.** $0.85 \cdot 0.05 + 0.15 \cdot 0.40 = 0.1025$, so 10.25% of the time

**c.** $\frac{0.85 \cdot 0.95}{0.85 \cdot 0.95 + 0.15 \cdot 0.60} \approx 0.900$, so about 90.0%

**d.** $\frac{0.15 \cdot 0.40}{0.85 \cdot 0.05 + 0.15 \cdot 0.40} \approx 0.585$, so about 58.5%

**47.** **a.** $\frac{673}{122 + 1617 + 528 + 673} \approx 0.229$, so about 22.9%

**b.** $\frac{212}{212 + 673} \approx 0.240$, so about 24%

**48.** $\frac{1}{9} \cdot \frac{1}{8} \cdot \frac{1}{7} \cdot \frac{1}{6} = \frac{1}{3024}$

**49.** $3 \cdot 2 \cdot 6 = 36$

**50.** **a.** $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$

**b.** $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

**c.** $6720 \cdot 120 = 806{,}400$

**51.** $2 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 1440$

**52.** Answers vary. Sample: Label the first 6 columns of a spreadsheet with the values of the coins. In the next row, use the spreadsheet random number function to generate a 1 (heads) or 0 (tails) in each cell. For each trial, a 7th column can total the product of each cell's coin value and random number. These cells can be copied down for $n = 100$ trials and the frequency of successful outcomes can be counted.

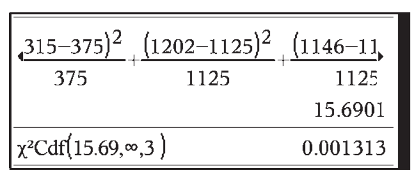
**53.** Answers vary. Sample: The experiment in Question 52 can be modified by changing the value in one of the penny columns to 5 and running another 100 trials.

**54.** Answers vary. Sample: Generate sets of 7 random numbers from 1–10 where 1 represents a rainy day. The number of 1's in a set of 7 numbers is the number of rainy days in a week. Do 100 trials to see the expected number of rainy days.

**55.** **a.** in the same order: 375, 1125, 1125, 3375

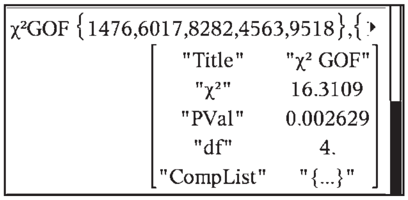
**b.** The number of degrees of freedom is 3.

**c.**

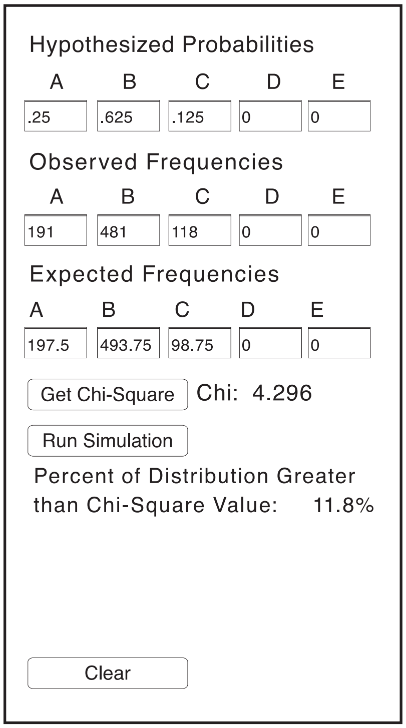



The probability of a chi-square value as high as 15.69 or greater is 0.0013. Because $0.0013 < 0.01$, we can reject the geneticist's claim.

**56.** The probability of a chi-square value as high as 16.31 or greater is 0.0026. Because $0.0026 > 0.001$ we do not have sufficient evidence to reject the claim of the hypothesized percentages.




**57.**




The chi-square simulation web applet gives a chi-square value of 4.296 and a probability of 11.8%. Because $0.118 > 0.05$, we have insufficient evidence to reject the claim of the hypothesized ratios of morning rush hour traffic.

**58. a.** $0.37 \cdot 0.43 + 0.29 \cdot 0.57 = 0.3244$, so 32.44%

**b.** $0.31 \cdot 0.43 + 0.37 \cdot 0.57 = 0.3442$, so 34.42%

**c.** Democrats: $0.3244 + 0.5 \cdot 0.34 \cdot 0.57 = 0.4213$; Republicans: $0.3442 + 0.5 \cdot 0.34 \cdot 0.57 = 0.4411$; If there is a tie, then the Democrats get 50% of the vote, $(0.32 \cdot 0.43)x + 0.4213 = 0.5$. Solving for $x$ shows that about 57.19% of the 18–35 Independents must vote Democrat for there to be a tie.

**59. a.** $\frac{42}{335} \approx 0.125$, so about 12.5%; $\frac{27}{206} \approx 0.131$, so about 13.1%

**b.** $\frac{21}{76} \approx 0.276$, so about 27.6%; $\frac{80}{277} \approx 0.289$, so about 28.9%

**c.** No, whether a student studies more or less than three hours, the relative frequency of a score increase of less than 25 points appears to be the same. The same is true for students whose scores increased more than 25 points.

**d.**

| | < 25 | > 25 | Total |
|---|---|---|---|
| **Less Study** | 107 | 376 | 483 |
| **More Study** | 63 | 348 | 411 |
| **Total** | 170 | 724 | 894 |

Score increase of < 25 points with less study: $\frac{107}{483} \approx 22.2\%$;
Score increase of < 25 points with more study: $\frac{63}{411} \approx 15.3\%$

**e.** These data exemplify Simpson's Paradox, since there appears to be a correlation between amount of study and performance when you combine the school districts, but not when you examine them separately.