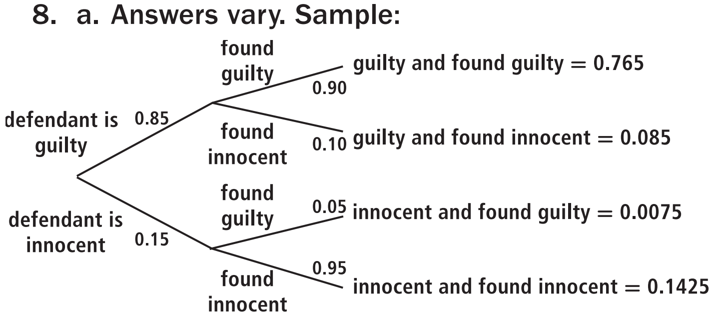
Chapter 6

Counting, Probability, and Inference

Self Test (pp. 430-431)

Questions

1. a. {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
   b. HTT, THT, TTT, TTH
2. a. $_{15}P_6 = \frac{15!}{(15-6)!} = \frac{15!}{9!} = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 = 3{,}603{,}600.$
   b. $_{15}P_6$ is the number of all permutations of 6 objects from 15 objects.
3. $_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$
4. Tossing the coin has 2 possible outcomes and rolling the die has 6 possible outcomes, so there are 12 possible outcomes in the sample space, by the Multiplication Counting Priciple. In five of these outcomes, the sum of the outcomes of the two events is less than 5: {1 + 3, 1 + 2, 1 + 1, 2 + 2, 2 + 1}. Therefore the probability is $\frac{5}{12}$.
5. a. The lengths of flosses in the packages are independent, so the probability is $(0.15)^4 \approx 0.0005$.
   b. The probability that a package has at least the advertised amount is $1 - 0.15 = 0.85$ because the two events are complements. The probability that all four packages have at least the stated amount is $(0.85)^4 \approx 0.5220$.
6. Use the Multiplication Counting Principle. $4 \cdot 5 \cdot 2 = 40$ different outfits.
7. a. By the Addition Counting Principle, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.8 - 0.6 = 0.9$
   b. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.6}{0.7} \approx 0.86$
8. a. Answers vary. Sample:



   b. $\frac{P(\text{innocent and found guilty})}{P(\text{found guilty})} = \frac{0.0075}{0.0075 + 0.765} \approx 0.97\%$
   c. $\frac{P(\text{guilty and found innocent})}{P(\text{found innocent})} = \frac{0.085}{0.085 + 0.1425} \approx 37.36\%$
9. true; If A and B are complementary, then they are mutually exclusive and thus make up the entire sample space. Since $P(S) = 1$, $P(A) + P(B) = 1$, and therefore $P(B) = 1 - P(A) = 1 - k$.
10. false; For a fair die, the probability of rolling a 2 is $\frac{1}{6}$. The expected number of "2"s in 600 rolls is $\frac{1}{6} \cdot 600 = 100$.
11. No. A and B are independent events if and only if $P(A \cap B) = P(A) \cdot P(B)$. Here, $P(A) \cdot P(B) = 0.21 \cdot 0.17 = 0.0357 \neq P(A \cap B) = 0.02$.
12. Calculate the expected distribution of majors and use this with the observed survey results to do the chi-square test: $\chi^2 = \frac{(125-156)^2}{156} + \frac{(312-351)^2}{351} + \frac{(138-117)^2}{117} + \frac{(205-156)^2}{156} \approx 29.65$. The probability of a χ^2 value of 29.65 or larger is less than 0.001, so the new survey yields sufficient evidence to reject the hypothesis that the proportions by major are still what the guidance office states.
13. By totaling the rows and columns we find that the total dorm population is 1020 and the total car-owning population is 290. $\frac{290}{1020} \approx 28.4\%$.
14. The total car-owning population is 290, and 70 juniors own cars. $\frac{70}{290} \approx 24.1\%$.
15. Since $.325 = \frac{13}{40}$, simulate this situation by generating random numbers between 1 and 40, where 1-13 represents a hit and 14-40 represents not a hit. Generating eight random numbers equals one trial. A trail is successful if 4 or more of the numbers generated are a hit (1-13). Run this simulation for forty trials and use the relative frequency of success to estimate the probability.
16. a. If the die is fair, the probability of rolling a 2 is $\frac{1}{6}$, so the expected count is $\frac{1}{6} \cdot 12{,}000 = 2{,}000$.
    b. Answers vary. Sample: You might conclude that the die is unfair because 1 occurs very frequently, and 4 occurs rarely. You could do a chi-square test to test your hypothesis.
    c. No, the sample is too small. By the Law of Large Numbers, the relative frequency of each of the outcomes should approach their probability as the number of trials increases. A chi-square test might be inaccurate on a sample this small.