# Chapter 13
## Further Work with Trigonometry
### Chapter Review (pp. 836–837)

1. $[\sqrt{34}, 30.96°]$; $r = \sqrt{5^2 + 3^2} = \sqrt{34}$; $\tan\theta = \frac{3}{5}$, $\theta = \tan^{-1}\left(\frac{3}{5}\right) \approx 30.96°$

2. $[\sqrt{13}, 163.90°]$; $r = \sqrt{(-2\sqrt{3})^2 + 1^2} = \sqrt{13}$; $\tan\theta \approx -0.289$, $\theta = \tan^{-1}(-0.289) \approx -16.12°$. $\theta$ is in the second quadrant so $\theta \approx -16.12° + 180° \approx 163.90°$.

3. $\left[\frac{1}{6}, 30°\right]$; $r = \frac{1}{6}$; $\frac{\pi}{6} = 30°$

4. $-\frac{5}{2} + \frac{5\sqrt{3}}{2}i$; $5(\cos(-240°) + i\sin(-240°)) = 5\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = -\frac{5}{2} + \frac{5\sqrt{3}}{2}i$

5. $-2.47 + 7.61i$; $a = 8\cos\frac{3\pi}{5} = -2(\sqrt{5} - 1) \approx -2.47$; $b = 8\sin\frac{3\pi}{5} = 2\sqrt{2(\sqrt{5} + 5)} \approx 7.61$

6. a. $[r, \theta] = [-3, 135°]$ is equivalent to $[3, 135° + 180°] = [3, 315°]$, so the number can be written in trigonometric form as $-3(\cos 135° + i\sin 135°)$, and $3(\cos 315° + i\sin 315°)$.

   b. $[r, \theta] = \left[7, \frac{\pi}{6}\right]$ is equivalent to $\left[7, \frac{\pi}{6} + 2\pi\right] = \left[7, \frac{13\pi}{6}\right]$, so the number can be written in trigonometric form as $7\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ and $7\left(\cos\frac{13\pi}{6} + i\sin\frac{13\pi}{6}\right)$.

   c. $r = \sqrt{2^2 + (-1)^2} = \sqrt{5}$; $\tan\theta = -\frac{1}{2}$, $\theta = \tan^{-1}\left(-\frac{1}{2}\right) \approx -26.57°$, which is equivalent to $-26.57° + 360° = 333.43°$, so the number can be written in trigonometric form as $\sqrt{5}(\cos -26.57° + i\sin -26.57°)$ and $\sqrt{5}(\cos 333.43° + i\sin 333.43°)$.

7. $\sqrt{(-9)^2 + 11^2} = \sqrt{202}$; $\tan\theta = -\frac{11}{9}$, $\theta = \tan^{-1}\left(-\frac{11}{9}\right) \approx -50.71$. Since $\theta$ is in the second quadrant, $\theta \approx 129.29$.

8. 12; the modulus is $r = 12$; $\frac{2\pi}{3}$; the argument is $\theta = \frac{2\pi}{3}$

9. $z_1z_2 = (4 \cdot 3)(\cos(23° + 175°) + i\sin(23° + 175°)) = 12(\cos 198° + i\sin 198°)$

10. $z_1z_2 = \left[6 \cdot 1.4, 0 + \frac{\pi}{3}\right] = \left[8.4, \frac{\pi}{3}\right]$

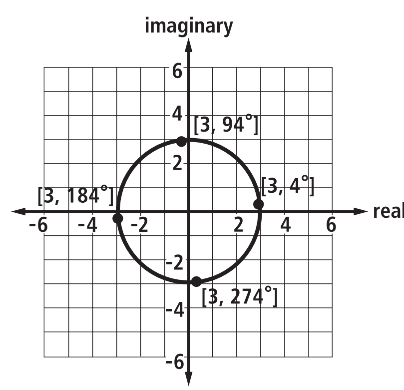
11. $\frac{z_1}{z_2} = \left[\frac{4}{5}, \frac{\pi}{8} - \frac{7\pi}{8}\right] = \left[\frac{4}{5}, -\frac{6\pi}{8}\right]$

12. $\frac{z_1}{z_2} = \frac{3}{6}\left(\cos\left(\frac{2\pi}{3} - \frac{5\pi}{6}\right) + i\sin\left(\frac{2\pi}{3} - \frac{5\pi}{6}\right)\right) = \frac{1}{2}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

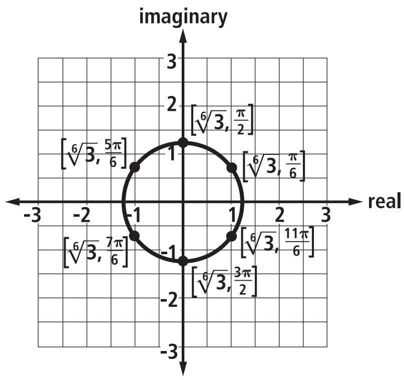
13. $z = \sqrt{2} + \sqrt{2}i = 2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$; $z^6 = 2^6\left(\cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4}\right) = 64(-1 + 0i) = -64i$

14. $z^5 = 2^5(\cos 1200° + i\sin 1200°) = 32(\cos 120° + i\sin 120°)$

15. $3(\cos 4° + i\sin 4°)$, $3(\cos 94° + i\sin 94°)$, $3(\cos 184° + i\sin 184°)$, $3(\cos 274° + i\sin 274°)$; The 4 fourth roots of $81(\cos 16° + i\sin 16°)$ are $81^{\frac{1}{4}}\left(\cos\left(\frac{16°}{4} + \frac{360°k}{4}\right) + i\sin\left(\cos\left(\frac{16°}{4} + \frac{360°k}{4}\right)\right)\right)$ for $k = 0, 1, 2, 3$.



**16.** $1.04 + 0.6i$, $1.2i$, $-1.04 + 0.6i$, $-1.04 - 0.6i$, $-1.2i$, $1.04 - 0.6i$; The 6 sixth roots of $-3 = 3(-1 + 0i) = 3(\cos \pi + i \sin \pi)$ are $3^{\frac{1}{6}}\left(\cos\left(\frac{\pi}{6} + \frac{2\pi k}{6}\right) + i \sin\left(\frac{\pi}{6} + \frac{2\pi k}{6}\right)\right)$ for $k = 0, 1, 2, 3, 4, 5$.




**17.** $(1 - \cos^2 x)(1 + \cot^2 x) = \sin^2 x(1 + \cot^2 x) = \sin^2 x + \sin^2 x \cot^2 x = \sin^2 x + \sin^2 x\left(\frac{\cos^2 x}{\sin^2 x}\right) = \sin^2 x + \cos^2 x = 1$

**18.** $\cos(\theta - \frac{\pi}{3}) = \cos\theta \cos\frac{\pi}{3} + \sin\theta \sin\frac{\pi}{3} = \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = \sin\frac{\pi}{6}\cos\theta + \cos\frac{\pi}{6}\sin\theta = \sin\left(\theta + \frac{\pi}{6}\right)$

**19.** $\csc^2 x - \cot^2 x = \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \cos^2 x}{\sin^2 x} = 1$

**20.** $\frac{\cos\theta}{\sin\theta \cdot \cot\theta} = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cot\theta} = \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = 1$ for $\theta \neq \frac{n\pi}{2}$, where $n$ is an integer.

**21.** $\sin x = 0$ for $x = n\pi$ where $n$ is an integer, $\sin x$ occurs in the denominator of the expression in Question 19, and division by 0 is not defined.

**22. a.** Since either $\sin x = 0$ or $\cot = x$ for all $x = \frac{n\pi}{2}$, where $n$ is an integer, these values of $x$ are singularities.

**b.** Answers vary. Sample: $0 < x < \frac{\pi}{2}$

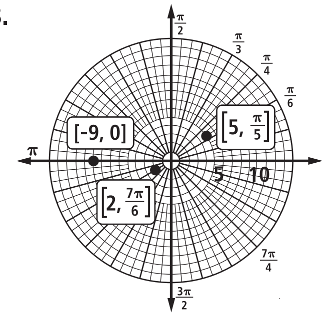
**23.** Since $(x - 4)$ is in the denominator, the function has a singularity for $x - 4 = 0$ which is $x = 4$.

**24.** none; The denominator cannot be divided into linear factors, so the function has no singularities.

**25. a.** false; It is true for all $x$, $x \neq 4$ because $x = 4$ is a singularity for the function.

**b.** true; $x^3 - 64 = (x - 4)(x^2 + 4x + 16)$, and the original expressions are both defined for all real numbers $x$.

**26.–28.**




**29.** B; $[3, 240°] = [3, 240° - 360°] = [3, -120°] = A = [-3, -120° + 180°] = [-3, 60°] = C = [3, 240° + 720°] = [3, 960°] = D \neq [-3, 120°]$

**30.** $(-4.24, 4.24)$; $x = 6 \cos\frac{3\pi}{4} = -3\sqrt{2} \approx -4.24$; $y = 6 \sin\frac{3\pi}{4} = 3\sqrt{2} \approx 4.24$

**31.** $(-5.73, -4.02)$; $x = -7 \cos 35° \approx -5.73$; $y = -7 \sin 35° \approx -4.02$

**32.** $(-0.90, -0.44)$; $x = -\cos 26° \approx -0.90$; $y = -\sin 26° \approx -0.44$

**33.** $r = \pm\sqrt{2^2 + 5^2} = \pm\sqrt{29}$; $\theta = \tan^{-1}\left(\frac{5}{2}\right) \approx 68.2°$, so two possible pairs of polar coordinates are $\left[\sqrt{29}, 68.2°\right]$ and $\left[-\sqrt{29}, 68.2° + 180°\right] = \left[-\sqrt{29}, 248.2°\right]$.

**34.** $r = \pm\sqrt{(-2)^2 + (-3)^2} = \pm\sqrt{13}$; $\theta = \tan^{-1}\left(\frac{2}{3}\right) \approx 33.69°$, so two possible pairs of polar coordinates are $\left[-\sqrt{13}, 33.69°\right]$ and $\left[\sqrt{13}, 33.69° + 180°\right] = \left[\sqrt{13}, 213.69°\right]$.

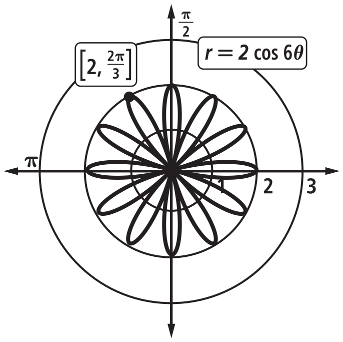
**35.** $r = \pm\sqrt{(-4)^2 + (-9)^2} = \pm\sqrt{97}$; $\theta = \tan^{-1}\left(-\frac{9}{4}\right) \approx 114°$, so two possible pairs of polar coordinates are $\left[\sqrt{97}, 114°\right]$ and $\left[\sqrt{97}, -246°\right]$.

**36.** $\left[6, \frac{5\pi}{6}\right], \left[-6, -\frac{\pi}{6}\right], \left[-6, \frac{11\pi}{6}\right]$;
$r = \pm\sqrt{(-3\sqrt{3})^2 + (3)^2} = \pm 6$;
$\theta = \tan^{-1}\left(-\frac{3}{3\sqrt{3}}\right) = \frac{5\pi}{6}$;
$[r, \theta] = \left[6, \frac{5\pi}{6}\right] = \left[-6, \frac{5\pi}{6} - \pi\right] = \left[-6, -\frac{\pi}{6}\right] = \left[-6, \frac{5\pi}{6} + \pi\right] = \left[-6, \frac{11\pi}{6}\right]$

Back to Chapter Review

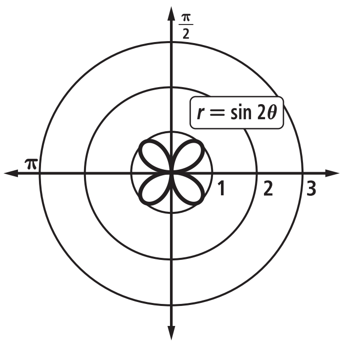
**37.** $(6, 4.36), \left[7.42, \frac{\pi}{5}\right]; 6 = r\cos\frac{\pi}{5}, r \approx 7.42,$
$y = r\cos\theta = 7.42\sin\frac{\pi}{5} \approx 4.36$

**38.**

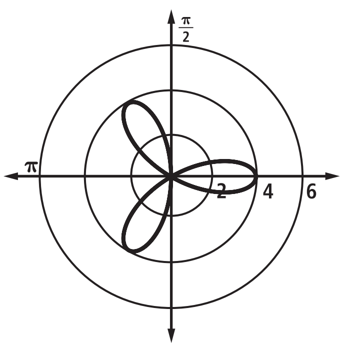



**39.** Answers vary. Sample: $\left[\sqrt{2}, \frac{\pi}{4}\right], \left[1, \frac{\pi}{2}\right];$
$\csc\frac{\pi}{4} = \sqrt{2}; \csc\frac{\pi}{2} = 1$

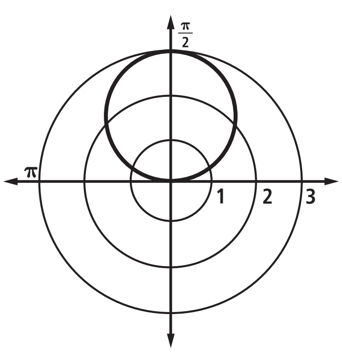
**40.**




**41.**

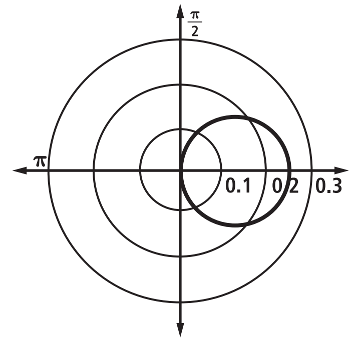



**42. a.**



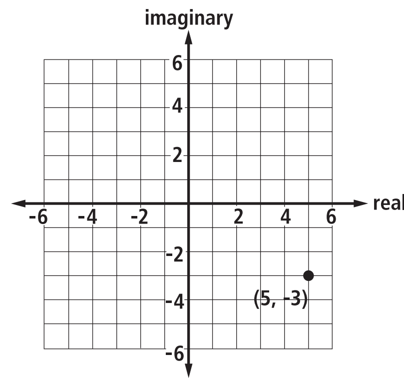

**b.** $x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$; The graph is a circle, so use the general equation for a circle with radius 1.5 and centered around $(0, 1.5)$.

**43. a.**

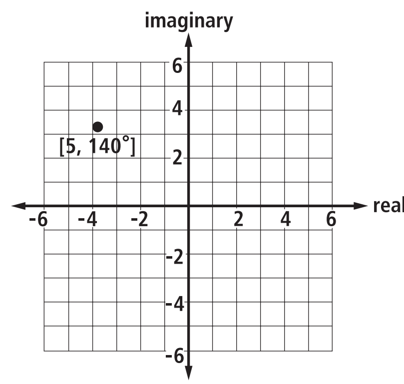



**b.** $\left(x - \frac{1}{8}\right)^2 + y^2 = \frac{1}{64}$; The graph is a circle, so use the general equation for a circle with radius $\frac{1}{8}$ and centered around $\left(\frac{1}{8}, 0\right)$.

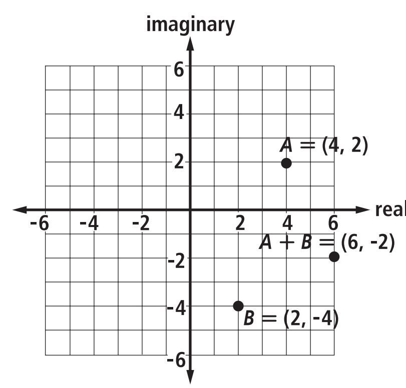
**44.**




**45.**




**46. a.**

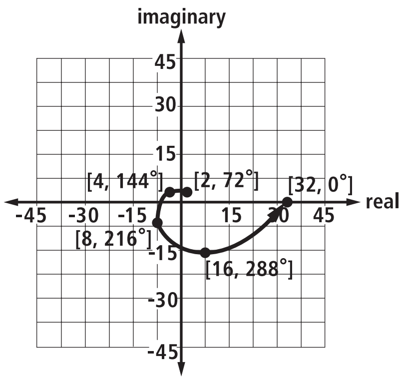


**b.** $m_{\overline{OA}} = \frac{2-0}{4-0} = \frac{1}{2} = m_{\overline{B(A+B)}}$;

$m_{\overline{OB}} = \frac{0-(-4)}{0-2} = -2 = m_{\overline{A(A+B)}}$.

Since the slopes of opposite sides are equal, the quadrilateral is a parallelogram.

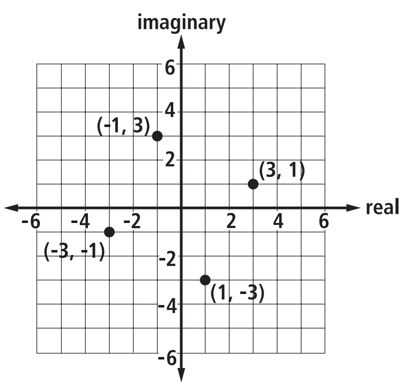
**47. a.**




**b.** They appear to form a logarithmic spiral.

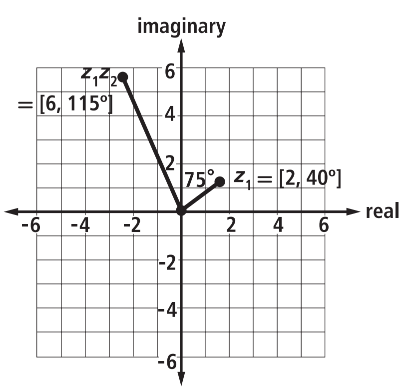
**c.** $z^5 = (2(\cos 72^\circ + i \sin 72^\circ))^5 =$
$2^5(\cos(5 \cdot 72^\circ) + i \sin(5 \cdot 72^\circ)) =$
$32(1 + 0i) = 32$

**48. a.**




**b.** They appear to form a square.

**49. a.**




**b.** a size-change of magnitude 3 and a rotation of magnitude 75°