c. Average rate $= \dfrac{d(t)-d(20)}{t-20} =$
$\dfrac{0.01t^2+0.5t-14}{t-20} = \dfrac{0.01(t+70)(t-20)}{t-20} =$
$0.01t+0.7$, $t \neq 20$. The limit as t approaches 20 is $0.01(20) + 0.7$, which equals 0.9 cm/day. This instantaneous rate is called the derivative.

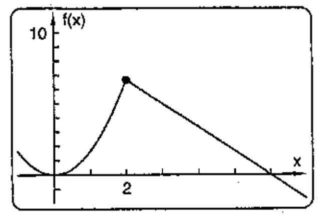
d. The glacier seems to be speeding up because each 10-day period it moved farther than it had in the preceding 10-day period.

T16. $c(0) = p(0) = 10$, so each has the same speed at $t = 0$. $\lim_{t\to\infty} c(t) = 16$. $\lim_{t\to\infty} p(t) = \infty$. Surprise for Phoebe!

T17. $f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 10-kx, & \text{if } x > 2 \end{cases}$

$\lim_{x\to 2^-} f(x) = k \cdot 2^2 = 4k$
$\lim_{x\to 2^+} f(x) = 10-2k$

Make $4k = 10 - 2k \Rightarrow k = 5/3$. There is a cusp at $x = 2$.



T18. $h(x) = x^3$. $h(1) = 1$ and $h(2) = 8$, so 7 is between $h(1)$ and $h(2)$. The intermediate value theorem allows you to conclude that there is a real number between 1 and 8 equal to the cube root of 7.

T19. Answers will vary.

Chapter Test

T1. f is continuous at $x = c$ if and only if
1. $f(c)$ exists
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

f is continuous on $[a, b]$ if and only if f is continuous at all points in (a, b), and $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$.

T2. a.
- $\lim_{x\to 2^-} f(x) = 3$
- $\lim_{x\to 2^+} f(x) = 4$
- $\lim_{x\to 2} f(x)$ does not exist.
- $\lim_{x\to 6^-} f(x) = 2$
- $\lim_{x\to 6^+} f(x) = 2$
- $\lim_{x\to 6} f(x) = 2$

b. f is continuous on [2, 6] because it is continuous for all values in (2, 6) and $\lim_{x\to 2^+} f(x) = f(2)$ and $\lim_{x\to 6^-} f(x) = f(6)$.

T3. See the text statement of the quotient property.

T4. a. Left: –4; right: –4

b. Limit: –4

c. Discontinuous

T5. a. Left: none; right: none

b. Limit: none

c. Discontinuous

T6. a. Left: 6; right: 6

b. Limit: 6

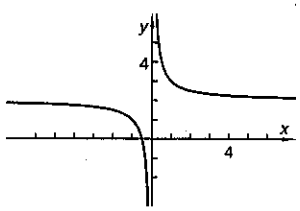
c. Continuous

T7. a. Left: –2; right: 3

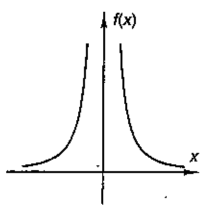
b. Limit: none

c. Discontinuous

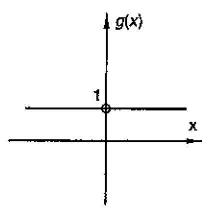
T8.



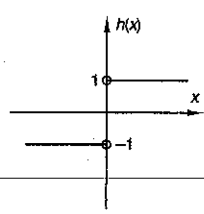
T9. a.



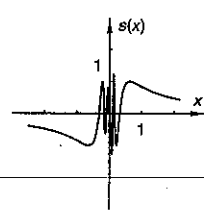
b.



c.



d.



T10. a. $f(3) = \dfrac{(0^2 - 5\cdot 0 + 8)(0-3)}{0-3} = \dfrac{0}{0}$, an indeterminate form

b. $\lim_{x\to 3} f(x) = \lim_{x\to 3} (x^2 - 5x + 8),\ x \neq 3$ — Definition of limit "$x \neq c$"

$= \lim_{x\to 3} x^2 + \lim_{x\to 3}(-5x) + \lim_{x\to 3} 8$ — Limit of a sum

$= \lim_{x\to 3} x \cdot \lim_{x\to 3} x + (-5)\cdot \lim_{x\to 3} x + 8$ — Limit of a product, limit of a constant times a function, limit of a constant

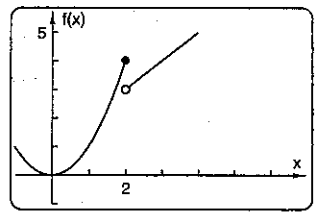
$= 3 \cdot 3 + (-5) \cdot 3 + 8$ — Limit of x

$= 2$, Q.E.D.

T11. If $k = 1$, $f(x) = \begin{cases} x^2, & x \le 2 \\ x+1, & x > 2 \end{cases}$

$\lim_{x\to 2^-} f(x) = 4$, $\lim_{x\to 2^+} f(x) = 3$

$\therefore f$ is discontinuous at $x = 3$.



T12. $\lim_{x\to 2^-} f(x) = k\cdot 2^2$

$\lim_{x\to 2^+} f(x) = 2 + k$

$\therefore 4k = 2 + k$

$k = 2/3$

T13. See graph in T11.

T14. a. $\lim_{x\to\infty} T(x) = 20$

From the graph, it appears that if $x > 63$ ft, then $T(x)$ is within 1° of the limit.

The graph of T has a horizontal asymptote at $T = 20$.

b. $T = 20 + 8(0.97^x)\cos 0.5x$. The amplitude of the cosine factor is $8(0.97^x)$. Make this amplitude < 0.1.

$8(0.97^c) = 0.1$

$0.97^c = 0.0125$

$c = \dfrac{\log 0.0125}{\log 0.97}$

$c = 143.8654\ldots$

$\therefore T$ is within 0.1 unit of 20 whenever $x > 143.8654\ldots$.

c. The time of day would be mid-afternoon, when the temperature of the surface is highest.

T15. a. Use either TRACE or TABLE to show: $d(0) = 0$, $d(10) = 6$, $d(20) = 14$, $d(30) = 24$, $d(40) = 36$, and $d(50) = 50$.

b. Average rate $= \dfrac{d(20.1) - d(20)}{20.1 - 20} = \dfrac{14.0901 - 14}{20.1 - 20} = 0.901$ cm/day