c. If $y = x^n$, then $y' = nx^{n-1}$.

d. See solution to Problem 35 in Problem Set 3-4.

e. See the proof in Section 3-4.

f. $\dfrac{dy}{dx}$ is pronounced "d y, d x."

$\dfrac{d}{dx}(y)$ is pronounced "d, d x, of y."

Both mean the derivative of y with respect to x.

g. i. $f(x) = 7x^{9/5} \Rightarrow f'(x) = \dfrac{63}{5}x^{4/5}$

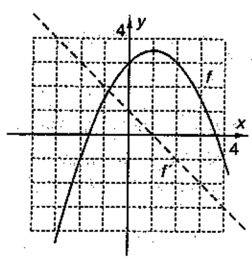
ii. $g(x) = 7x^{-4} - \dfrac{x^2}{6} - x + 7 \Rightarrow$

$g'(x) = -28x^{-5} - \dfrac{x}{3} - 1$

iii. $h(x) = 7^3 \Rightarrow h'(x) = 0$

h. $f'(32) = \frac{63}{5}(32)^{4/5} = 201.6$ exactly. The numerical derivative is equal to or very close to 201.6.

i.



R5. a. $v = \dfrac{dx}{dt}$ or $x'(t)$.

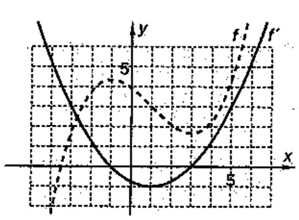
$a = \dfrac{dv}{dt}$ or $v'(t)$, $a = \dfrac{d^2x}{dt^2}$ or $x''(t)$

b. $\dfrac{d^2y}{dx^2}$ means the second derivative of y with respect to x.

$y = 10x^4 \Rightarrow y' = 40x^3 \Rightarrow y'' = 120x^2$

c. $f'(x) = 12x^3 \Rightarrow f(x) = 3x^4 + C$. $f(x)$ is the antiderivative, or the indefinite integral, of $f(x)$.

d. The slope of $y = f(x)$ is determined by the value of $f'(x)$. So the slope of $y = f(x)$ at $x = 1$ is $f'(1) = -1$, at $x = 5$ is $f'(5) = 3$, and at $x = -1$ is $f'(-1) = 0$.



e. i. $y = -0.01t^3 + 0.9t^2 - 25t + 250$

$v = \dfrac{dy}{dt} = -0.03t^2 + 1.8t - 25$

$a = \dfrac{dv}{dt} = -0.06t + 1.8$

ii. $a(15) = -0.06(15) + 1.8 = 0.9$ (km/s)/s

$v(15) = -0.03(15^2) + 1.8(15) - 25$
$= -4.75$ km/s

The spaceship is slowing down at $t = 15$ because the velocity and the acceleration have opposite signs.

iii. $v = -0.03t^2 + 1.8t - 25 = 0$

By using the quadratic formula or the solver feature of your grapher, $t = 21.835\ldots$ or $t = 38.164\ldots$.

The spaceship is stopped at about 21.8 and 38.2 seconds.

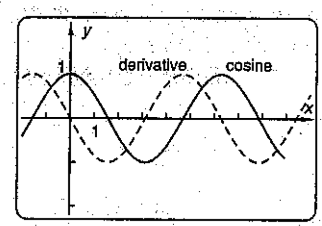
iv. $y = -0.01t^3 + 0.9t^2 - 25t + 250 = 0$

By using TRACE or the solver feature of your grapher, $t = 50$.

$v(50) = -10$

Because the spaceship is moving at 10 km/s when it reaches the surface, it is a crash landing!

R6. a.



b. The graph of the derivative is the same as the sine graph but inverted in the y-direction. Thus, $(\cos x)' = -\sin x$ is confirmed.

c. $-\sin 1 = -0.841470984\ldots$

Numerical derivative $\approx -0.841470984\ldots$

The two are very close!

d. Composite function

$f'(x) = -2x \sin(x^2)$

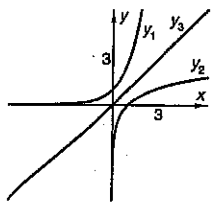
R7. a. i. $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

ii. $f(x) = g(h(x)) \Rightarrow f'(x) = g'(h(x)) \cdot h'(x)$

iii. The derivative of a composite function is the derivative of the outside function with respect to the inside function times the derivative of the inside function with respect to x.

b. See the derivation in the text. This derivation constitutes a proof. Δu must be nonzero throughout the interval.

d.



$y_1 = e^x$ is the inverse of $y_2 = \ln x$, so y_1 is a reflection of y_2 across the line $y = x$.

$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Concept Problems

C1. a. $f(x) = x^7$, $g(x) = x^9$. So $h(x) = f(x) \cdot g(x) = x^{16}$.

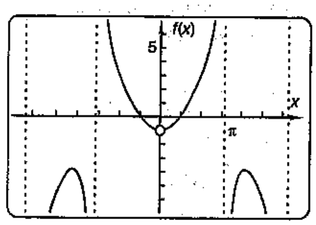
b. $h'(x) = 16x^{15}$

c. $f'(x) = 7x^6$, $g'(x) = 9x^8$. So $f'(x) \cdot g'(x) = 63x^{14} \neq h'(x)$.

d. $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) = 7x^6 \cdot x^9 + x^7 \cdot 9x^8 = 16x^{15}$

C2. a. $f(x) = \dfrac{x - \sin 2x}{\sin x}$. $f(0)$ has the form 0/0, which is indeterminate. f is discontinuous at $x = 0$ because $f(0)$ does not exist.

b. By graph (below) or by TABLE, $f(x)$ seems to approach -1 as x approaches zero. Define $f(0)$ to be -1.



c. Conjecture: The function *is* differentiable at $x = 0$. The derivative should equal zero because the graph is horizontal at $x = 0$.

d. $$f'(0) = \lim_{h \to 0} \frac{f(x) - f(0)}{x - 0}$$
$$= \lim_{x \to 0} \frac{\dfrac{x - \sin 2x}{\sin x} - (-1)}{x}$$
$$= \lim_{x \to 0} \frac{x - \sin 2x + \sin x}{x \sin x}$$

Using TABLE for numerator, denominator, and quotient shows that the numerator goes to zero faster than the denominator. For instance, if $x = 0.001$,

$$\text{quotient} = \frac{1.1666\ldots \times 10^{-9}}{9.999\ldots \times 10^{-7}} = 0.00116\ldots$$

Thus, the limit appears to be zero. (The limit can be found algebraically to equal zero by l'Hospital's rule after students have studied Section 6-5.)

Chapter Test

T1. See the definition of derivative in Section 3-2 or 3-4.

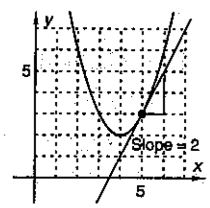
T2. Prove that if $f(x) = 3x^4$, then $f'(x) = 12x^3$.

Proof:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^4 - 3x^4}{h}$$
$$= \lim_{h \to 0} \frac{3x^4 + 12x^3h + 18x^2h^2 + 12xh^3 + 3h^4 - 3x^4}{h}$$
$$= \lim_{h \to 0} (12x^3 + 18x^2h + 12xh^2 + 3h^3) = 12x^3,$$

Q.E.D.

T3. If you zoom in on the point where $x = 5$, the graph appears to get closer and closer to the tangent line. The name of this property is *local linearity*.



T4. Amos substituted *before* differentiating instead of *after*. Correct solution is $f(x) = 7x \Rightarrow f'(x) = 7 \Rightarrow f'(5) = 7$.

T5. $f(x) = (7x + 3)^{15} \Rightarrow f'(x) = 105(7x + 3)^{14}$

T6. $g(x) = \cos(x^5) \Rightarrow g'(x) = -5x^4 \sin x^5$

T7. $\dfrac{d}{dx}[\ln(\sin x)] = \dfrac{1}{\sin x} \cdot \cos x = \cot x$

T8. $y = 3^{6x} \Rightarrow y' = (\ln 3)3^{6x}(6) = 6(\ln 3)3^{6x}$

T9. $f(x) = \cos(\sin^5 7x) \Rightarrow$

$f'(x) = -\sin(\sin^5 7x) \cdot 5\sin^4 7x \cdot \cos 7x \cdot 7$

$= -35 \sin(\sin^5 7x) \sin^4 7x \cos 7x$

T10. $y = 60x^{2/3} - x + 2^5 \Rightarrow y' = 40x^{-1/3} - 1$

T11. $y = e^{9x} \Rightarrow \dfrac{dy}{dx} = 9e^{9x} \Rightarrow \dfrac{d^2y}{dx^2} = 81e^{9x}$

T12. $y' \approx 0.6$ (Function is $y = -3 + 1.5^x$, for which the numerical derivative is 0.6081... .)

T13. $y = 3 + 5x^{-1.6}$

$v(x) = 5(-1.6)x^{-2.6} = -8x^{-2.6}$

$a(x) = -8(-2.6)x^{-3.6} = 20.8x^{-3.6}$

Acceleration is the second derivative of the displacement function.

T14. $f'(x) = 72x^{5/4} \Rightarrow f(x) = 32x^{9/4}$

T15. $f'(x) = 5\sin x$ and $f(0) = 13$

$f(x) = -5\cos x + C$

$13 = -5\cos 0 + C \Rightarrow C = 18$

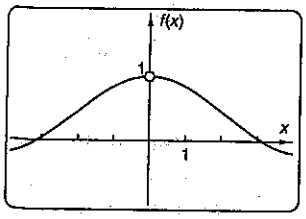
$f(x) = -5\cos x + 18$

T16. $f(x) = \cos 3x \Rightarrow f'(x) = -3\sin 3x$

$f'(5) = -3\sin 15 = -1.95086\ldots$

Decreasing at 1.95... y-units per x-unit.

T17. $f(x) = \dfrac{\sin x}{x}$



As x approaches zero, $f(x)$ approaches 1.
The squeeze theorem states:

If (1) $g(x) \le h(x)$ for all x in a neighborhood of c, (2) $\lim\limits_{x\to c} g(x) = \lim\limits_{x\to c} h(x) = L$, and (3) f is a function for which $g(x) \le f(x) \le h(x)$ for all x in that neighborhood of c, then $\lim\limits_{x\to c} f(x) = L$.

T18.

h	$\dfrac{5^h - 1}{h}$
-0.0003	1.6090...
-0.0002	1.6091...
-0.0001	1.6093...
0	undefined
0.0001	1.6095...
0.0002	1.6097...
0.0003	1.6098...

$\ln 5 = 1.6094\ldots$. The table shows that $\lim\limits_{h\to 0} \dfrac{5^h - 1}{h} = \ln 5$.

Proof:

from table

$\dfrac{d}{dx}(5^x) = \lim\limits_{h\to 0} \dfrac{5^{x+h} - 5^x}{h}$ Definition of derivative.

$= 5^x \lim\limits_{h\to 0} \dfrac{5^h - 1}{h}$ Factor out 5^x.

$= 5x \cdot \ln 5$ Evaluate.

T19. $v(t) = 251(1 - 0.88^t)$

$a(t) = 251[-\ln(0.88)]\,0.88^t = -251(\ln 0.88)0.88t$

$a(10) = -251(\ln 0.88)(0.88)(10) = 8.9360\ldots$

Numerical derivative gives 8.9360... as well.

T20. If the velocity and the acceleration have opposite signs for a particular value of t, then the object is slowing down at that time.

T21. a. $v(t) = t^{1.5} + 3 \Rightarrow a(t) = 1.5t^{0.5}$

b. $d(t) = \left(\dfrac{t^{2.5}}{2.5}\right) + 3t + C$

$d(1) = 20$

$\dfrac{1^{2.5}}{2.5} + 3(1) + C = 20$

$3.4 + C = 20$

$C = 16.6$

$\therefore d(t) = 0.4t^{2.5} + 3t + 16.6$

c. $d(9) - d(1) = 120.8$

This represents the displacement between the first and ninth seconds.

T22. a. $c(t) = 300 + 2\cos\dfrac{2\pi}{365}t \Rightarrow$

$c'(t) = -\dfrac{4\pi}{365}\sin\dfrac{2\pi}{365}t$

b. $c'(273) = -\dfrac{4\pi}{365}\sin\left(\dfrac{2\pi}{365}\cdot 273\right)$

$= 0.03442\ldots$ ppm/day

c. Rate is $(6\times 10^{15})\cdot\dfrac{0.03442\ldots}{1{,}000{,}000}\cdot\dfrac{1}{24\cdot 60\cdot 60} =$ $2390.6627\ldots$, which is approximately 2390 tons per second!

T23. Answers will vary.