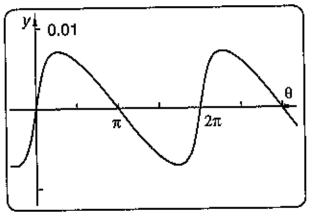
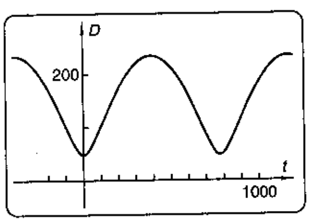
e. To maximize $\frac{dD}{dt}$, plot the variable part of $\frac{dD}{dt}$, $y = \frac{\sin\theta}{\sqrt{28530 - 26226\cos\theta}}$.



From the graph, it is clear that the maximum occurs well before $\theta = \pi/2$ (90°). Using the maximize feature, the maximum occurs at $\theta \approx 0.8505\ldots$, or $48.7\ldots°$.
(The exact value is $\cos^{-1}(93/141)$. One can find this by finding $(dl/dt)(dD/dt)$ and setting it equal to zero. One can also see this by decomposing Earth's motion vector into two components—one toward/away from Mars and the other perpendicular to the first. The rate of change in D is maximized when all of Earth's motion is along the Earth-to-Mars component, which occurs when the Earth-Mars-Sun triangle has a right angle at Earth. In this case, $\cos\theta = \frac{93}{141}$.)

f. $\theta = \left(\frac{1}{365} - \frac{1}{687}\right)2\pi t$ if t = days since 27 Aug. 2003.

$$D = \sqrt{28530 - 26226\cos\left[\left(\frac{1}{365} - \frac{1}{687}\right)2\pi t\right]}$$



The graph is not a sinusoid. The high and low points are not symmetric.

14. As B moves from negative values of x to positive values of x, the length of AB decreases to about 0.56 unit, then begins to increase when the x-value of point B passes about -0.3.
Let l = length of AB.
Know: $\frac{dx}{dt} = 2$ units/s. Want: $\frac{dl}{dt}$.
$l = \sqrt{e^{0.8x} + x^2} \Rightarrow \frac{dl}{dt}$
$= \frac{1}{2}(e^{0.8x} + x^2)^{-1/2} \cdot \left(2x\frac{dx}{dt} + 0.8e^{0.8x}\frac{dx}{dt}\right)$
$= \frac{0.8e^{0.8x} + 2x}{\sqrt{e^{0.8x} + x^2}}$
$\frac{dl}{dt} = -1.9963\ldots$ units/s at $x = -5$ units.
$\frac{dl}{dt} = 2.6610\ldots$ units/s at $x = 2$ units.
The length of AB is at a minimum when $dl/dt = 0$.
Use your grapher to solve $0.8e^{0.8x} + 2x = 0$.
At $x = -0.3117\ldots$, the length of AB stops decreasing and starts increasing.

Problem Set 4-10

Review Problems

R0. Answers will vary.

R1. a. $x = g(t) = t^3 \Rightarrow g'(t) = 3t^2$
$y = h(t) = \cos t \Rightarrow h'(t) = -\sin t$
If $f(t) = g(t) \cdot h(t) = t^3\cos t$, then, for example, $f'(1) = 0.7794\ldots$ by numerical differentiation.
$g'(1) \cdot h'(1) = 3(1^2) \cdot (-\sin 1) = -2.5244\ldots$
$\therefore f'(t) \neq g'(t) \cdot h'(t)$, Q.E.D.

b. If $f(t) = g(t)/h(t) = t^3/\cos t$, then, for example, $f'(1) = 8.4349\ldots$ by numerical differentiation.
$g'(1)/h'(1) = 3(1^2)/(-\sin 1) = 3.5651\ldots$
$\therefore f'(t) \neq g'(t)/h'(t)$, Q.E.D.

c. $y = \cos t$
$x = t^3 \Rightarrow t = x^{1/3} \Rightarrow y = \cos(x^{1/3})$
$\frac{dy}{dx} = -\sin(x^{1/3}) \cdot \frac{1}{3}x^{-2/3}$
At $x = 1$, $\frac{dy}{dx} = -\sin 1 \cdot \frac{1}{3} = -0.280490\ldots$.
If $x = 1$, then $t = 1^{1/3} = 1$.
$\therefore \frac{dy/dt}{dx/dt} = \frac{-\sin t}{3t^2} = \frac{-\sin 1}{3} = -0.280490\ldots$,
which equals dy/dx, Q.E.D.

R2. a. If $y = uv$, then $y' = u'v + uv'$.

b. See the proof of the product formula in the text.

c. i. $f(x) = x^7 \ln 3x \Rightarrow$
$f'(x) = 7x^6 \ln 3x + x^7 \cdot \frac{3}{3x} = 7x^6 \ln 3x + x^6$

ii. $g(x) = \sin x \cos 2x \Rightarrow$
$g'(x) = \cos x \cos 2x - 2\sin x \sin 2x$

iii. $h(x) = (3x - 7)^5(5x + 2)^3$
$h'(x) = 5(3x - 7)^4(3) \cdot (5x + 2)^3$
$+ (3x - 7)^5(3)(5x + 2)^2(5)$
$= 15(3x - 7)^4(5x + 2)^2(5x + 2$
$+ 3x - 7)$
$= 15(3x - 7)^4(5x + 2)^2(8x - 5)$

iv. $s(x) = x^8e^{-x} \Rightarrow s'(x) = -x^8e^{-x} + 8x^7e^{-x}$

d. $f(x) = (3x + 8)(4x + 7)$

i. $f'(x) = 3(4x + 7) + (3x + 8)(4) = 24x + 53$

ii. $f(x) = 12x^2 + 53x + 56$
$f'(x) = 24x + 53$, which checks.

R3. a. If $y = u/v$, then $y' = \dfrac{u'v - uv'}{v^2}$.

b. See proof of quotient formula in text.

c. i. $f(x) = \dfrac{\sin 10x}{x^5} \Rightarrow$

$$f'(x) = \frac{10\cos 10x \cdot x^5 - \sin 10x \cdot 5x^4}{x^{10}}$$
$$= \frac{10x\cos 10x - 5\sin 10x}{x^6}$$

ii. $g(x) = \dfrac{(2x+3)^9}{(9x-5)^4} \Rightarrow g'(x)$

$$= \frac{9(2x+3)^8 \cdot 2(9x-5)^4 - (2x+3)^9 \cdot 4(9x-5)^3 \cdot 9}{(9x-5)^8}$$
$$= \frac{18(2x+3)^8(5x-11)}{(9x-5)^5}$$

iii. $h(x) = (100x^3 - 1)^{-5} \Rightarrow$

$$h'(x) = -5(100x^3 - 1)^{-6} \cdot 300x^2$$
$$= -1500x^2(100x^3 - 1)^{-6}$$

d. $y = 1/x^{10}$

As a quotient:

$$y' = \frac{0 \cdot x^{10} - 1 \cdot 10x^9}{x^{20}} = \frac{-10}{x^{11}} = -10x^{-11}$$

As a power:

$y = x^{-10}$

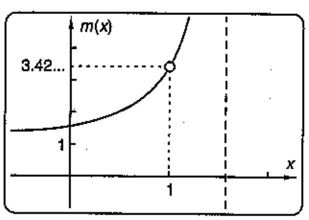
$y' = -10x^{-11}$, which checks.

e. $t(x) = \dfrac{\sin x}{\cos x} = \tan x$

$$t'(x) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$t'(1) = \sec^2 1 = 3.4255\ldots$

f. $m(x) = \dfrac{t(x) - t(1)}{x - 1} = \dfrac{\tan x - \tan 1}{x - 1}$



x	$m(x)$
0.997	3.40959...
0.998	3.41488...
0.999	3.42019...
1	undefined
1.001	3.43086...
1.002	3.43622...
1.003	3.44160...

The values get closer to 3.4255... as x approaches 1 from either side, Q.E.D.

R4. a. i. $y = \tan 7x \Rightarrow y' = 7\sec^2 7x$

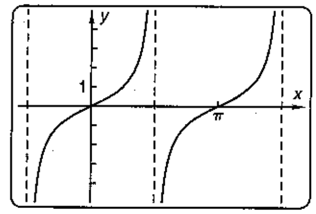
ii. $y = \cot(x^4) \Rightarrow y' = -4x^3 \csc^2(x^4)$

iii. $y = \sec e^x \Rightarrow y' = e^x \sec e^x \tan e^x$

iv. $y = \csc x \Rightarrow y' = -\csc x \cot x$

b. See derivation in text for $\tan' x = \sec^2 x$.

c. The graph is always sloping upward, which is connected to the fact that $\tan' x$ equals the square of a function and is thus always positive.



d. $f(t) = 7\sec t \Rightarrow f'(t) = 7\sec t \tan t$

$f'(1) = 20.17\ldots$

$f'(1.5) = 1395.44\ldots$

$f'(1.57) = 11038634.0\ldots$

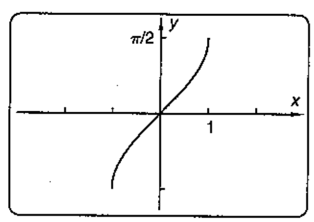
There is an asymptote in the secant graph at $t = \pi/2 = 1.57079\ldots$. As t gets closer to this value, secant changes very rapidly!

R5. a. i. $y = \tan^{-1} 3x \Rightarrow y' = \dfrac{3}{1 + 9x^2}$

ii. $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2 - 1}}$

iii. $c(x) = (\cos^{-1} x)^2 \Rightarrow c'(x) = \dfrac{-2\cos^{-1} x}{\sqrt{1 - x^2}}$

b. $y = \sin^{-1} x \Rightarrow y' = \dfrac{1}{\sqrt{1 - x^2}}$



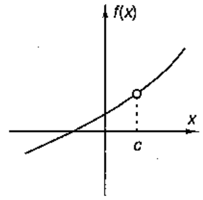
$y'(0) = \dfrac{1}{\sqrt{1 - 0^2}} = 1$, which agrees with the graph.

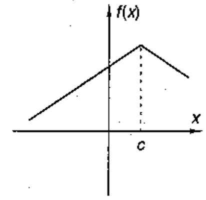
$y'(1) = \dfrac{1}{\sqrt{1 - 1^2}} = \dfrac{1}{0}$, which is infinite.

The graph becomes vertical as x approaches 1 from the negative side. $y'(2)$ is undefined because $y(2)$ is not a real number.

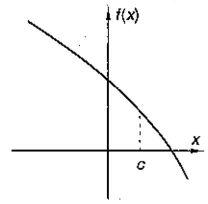
R6. a. Differentiability implies continuity.

b. i. Answers may vary. ii. Answers may vary.

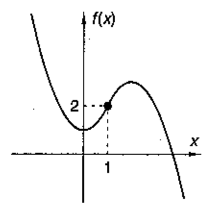




iii. No such function. iv. Answers may vary.



c. i.



ii. f is continuous at $x = 1$ because right and left limits both equal 2, which equals $f(1)$.

iii. f is differentiable. Left and right limits of $f'(x)$ are both equal to 2, and f is continuous at $x = 2$.

d. $g(x) = \begin{cases} \sin^{-1} x, & \text{if } 0 \le x \le 1 \\ x^2 + ax + b, & \text{if } x \le 0 \end{cases}$

$g'(x) = \begin{cases} (1-x)^{-1/2}, & \text{if } 0 < x < 1 \\ 2x + a, & \text{if } x < 0 \end{cases}$

$\lim_{x \to 0^-} g(x) = 0 + 0a + b = b$

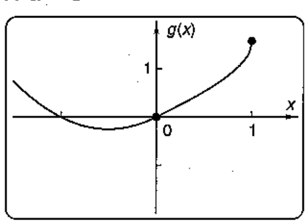
$\lim_{x \to 0^+} g(x) = \sin^{-1} 0 = 0$

$\therefore b = 0$

$\lim_{x \to 0^-} g'(x) = 0 + a = a$

$\lim_{x \to 0^+} g'(x) = 1^{-1/2} = 1$

$\therefore a = 1$



The graph appears to be differentiable and continuous at $x = 0$.

R7. a. $x = e^{2t}, y = t^3 \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3t^2}{2e^{2t}} \Rightarrow$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{3t^2}{2e^{2t}}\right)$

$= \dfrac{6t(dt/dx) \cdot 2e^{2t} - 3t^2 \cdot 4e^{2t}(dt/dx)}{(2e^{2t})^2}$

$= \dfrac{3t - 3t^2}{e^{2t}} \div \dfrac{dx}{dt} = \dfrac{3t - 3t^2}{2e^{4t}}$

b. $x = (t/\pi) \cos t$

$y = (t/\pi) \sin t$

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{(1/\pi)\sin t + (t/\pi)(\cos t)}{(1/\pi)\cos t + (t/\pi)(-\sin t)}$

$= \dfrac{\sin t + t \cos t}{\cos t - t \sin t}$

Where the graph crosses the positive x-axis, $t = 0, 2\pi, 4\pi, 6\pi, \ldots$.

If $t = 6\pi$, $x = 6$ and $y = 0$.

$\therefore$ (6, 0) *is* on the graph.

If $t = 6\pi$, then

$\dfrac{dy}{dx} = \dfrac{\sin 6\pi + 6\pi \cos 6\pi}{\cos 6\pi - 6\pi \sin 6\pi} = \dfrac{0 + 6\pi}{1 - 0} = 6\pi.$

So the graph is *not* vertical where it crosses the x-axis. It has a slope of $6\pi = 18.84\ldots$.

c. At a high point, y is a maximum and x is zero.

Use cosine for y and sine for x.

For y, the sinusoidal axis is at 25 ft.

For x, the sinusoidal axis is at 0 ft.

Both x and y have amplitude 20 ft, the radius of the Ferris wheel.

The phase displacement is 3 seconds.

The period is 20 seconds, so the coefficient of the arguments of sine and cosine is $2\pi/20 = \pi/10$.

$x = 20 \sin \dfrac{\pi}{10}(t - 3)$

$y = 25 + 20 \cos \dfrac{\pi}{10}(t - 3)$

$dx/dt = 2\pi \cos \dfrac{\pi}{10}(t - 3)$

$dy/dt = -2\pi \sin \dfrac{\pi}{10}(t - 3)$

When $t = 0$, $dy/dt = 5.0832\ldots$.

The Ferris wheel is going up at about 5.1 ft/s.

When $t = 0$, $dx/dt = 3.6931\ldots$.

The Ferris wheel is going right at about 3.7 ft/s.

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

dy/dx will be infinite if $dx/dt = 0$ and $dy/dt \ne 0$.

$dx/dt = 0$ if $2\pi \cos \dfrac{\pi}{10}(t - 3) = 0$.

$\dfrac{\pi}{10}(t - 3) = \dfrac{\pi}{2} + \pi n$ (where n is an integer)

$t = 8 + 5n$

The first positive time is $t = 8$ s.

R8. a. $y = x^{8/5} \Rightarrow y^5 = x^8$

$5y^4y' = 8x^7 \Rightarrow y' = \dfrac{8x^7}{5y^4} = \dfrac{8x^7}{5(x^{8/5})^4} = \dfrac{8}{5}x^{3/5}$

Using the power rule directly:

$y = x^{8/5} \Rightarrow y' = \frac{8}{5}x^{3/5}$

b. $y^3 \sin xy = x^{4.5} \Rightarrow$

$3y^2y' \cdot \sin xy + y^3(\cos xy)(y + xy') = 4.5x^{3.5}$

$y'[3y^2 \sin xy + xy^3 \cos xy]$

$= 4.5x^{3/5} - y^4 \cos xy$

$y' = \dfrac{dy}{dx} = \dfrac{4.5x^{3.5} - y^4 \cos xy}{3y^2 \sin xy + xy^3 \cos xy}$

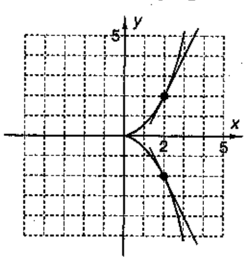
c. i. $4y^2 - xy^2 = x^3 \Rightarrow$

$8yy' - y^2 - x \cdot 2yy' = 3x^2$

$y'(8y - 2xy) = 3x^2 + y^2$

$y' = \dfrac{dy}{dx} = \dfrac{3x^2 + y^2}{8y - 2xy}$

At (2, 2), $dy/dx = 2$. At (2, −2), $dy/dx = -2$. Lines at these points with these slopes are tangent to the graph (see diagram).



ii. At (0, 0), dy/dx has the indeterminate form 0/0, which is consistent with the cusp.

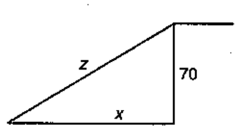
iii. To find the asymptote, solve for y.

$(4 - x)y^2 = x^3$

$y^2 = \dfrac{x^3}{4 - x}$

As x approaches 4 from the negative side, y becomes infinite. If $x > 4$, y^2 is negative, and thus there are no real values of y. Asymptote is at $x = 4$.

R9.



Let x = Rover's distance from the table.

Let z = slant length of tablecloth.

Know: $\dfrac{dx}{dt} = 20$ cm/s. Want: $\dfrac{dz}{dt}$ at $z = 200$.

$z^2 = x^2 + 70^2$

$2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt}$

$\dfrac{dz}{dt} = \dfrac{x}{z}\dfrac{dx}{dt} = \dfrac{20x}{z}$

At $z = 200$, $x = \sqrt{200^2 - 70^2} = 30\sqrt{39}$

$\dfrac{dz}{dt} = \dfrac{20 \cdot 30\sqrt{39}}{200} = 3\sqrt{39} = 18.7349...$

The glass moves at the same speed as the tablecloth, or about 18.7 cm/s, which is about 1.3 cm/s slower than Rover.

Concept Problems

C1. a. Let (x, y) be the coordinates of a point on the tangent line.

$\dfrac{y - y_0}{x - x_0} = m \Rightarrow y = m(x - x_0) + y_0$

b. Substituting $(x_1, 0)$ for (x, y) gives

$0 = m(x_1 - x_0) + y_0 \Rightarrow x_1 = x_0 - \dfrac{y_0}{m}$, Q.E.D.

c. The tangent line intersects the x-axis at $(x_2, 0)$. Repeating the above reasoning with x_2 and x_1 in place of x_1 and x_0 gives

$x_2 = x_1 - \dfrac{y_1}{m}$

Because $y_1 = f(x_1)$ and $m = f'(x_1)$,

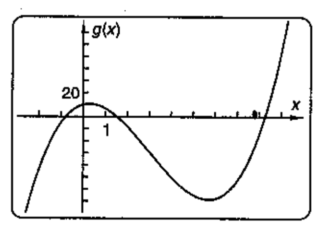
$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$, Q.E.D.

d. Programs will vary according to the kind of grapher used. The following steps are needed:

- Store $f(x)$ in the Y= menu.
- Input a starting value of x.
- Find the new x using the numerical derivative.
- Display the new x.
- Save the new x as the old x and repeat.

For $f(x) = x^2 - 9x + 14$, the program should give $x = 2$, $x = 7$.

e. For $g(x) = x^3 - 9x^2 + 5x + 10$, first plot the graph to get approximations for the initial values of x.



Run the program three times with $x_0 = -1$, 1, and 8. The values of x are

$x = -0.78715388...$

$x = 1.54050386...$

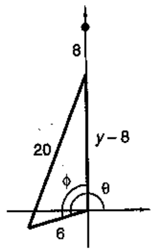
$x = 8.24665002...$

The answers are the same using the built-in solver feature. The same preliminary analysis is needed to find starting values of x.

f. $f(x) = \sec x - 1.1$

Starting with $x_0 = 1$, it takes seven iterations to get $x = 0.429699666\ldots$.

C2. a. The connecting rod, the crankshaft, and the y-axis form a triangle with angle $\phi = \theta - \pi/2$ included between sides of 6 cm and $(y - 8)$ cm.



By the law of cosines,

$$20^2 = (y-8)^2 + 6^2 - 2 \cdot 6 \cdot (y-8)\cos(\theta - \pi/2)$$

$$20^2 = (y-8)^2 + 6^2 - 12(y-8)\sin\theta$$

$$(y-8)^2 - 12\sin\theta\,(y-8) - 364 = 0$$

Solve for $y - 8$ using the quadratic formula.

$y - 8 = 6\sin\theta + \sqrt{(36\cdot\sin^2\theta + 364)}$. (The solution with the negative radical gives a triangle below the origin, which has no real-life meaning.)

$$y = 8 + 6\sin\theta + 2\sqrt{9\cdot\sin^2\theta + 91}$$

b. $v = \dfrac{dy}{dt} = 6\cos\theta\dfrac{d\theta}{dt} + \dfrac{18\sin\theta\cos\theta}{\sqrt{9\cdot\sin^2\theta+91}}\dfrac{d\theta}{dt}$

$$v = 6\cos\theta\frac{d\theta}{dt} + \frac{9\sin 2\theta}{\sqrt{9\cdot\sin^2\theta+91}}\frac{d\theta}{dt}$$

c. $a = \dfrac{d^2y}{dt^2}$

$$= -6\sin\theta\left(\frac{d\theta}{dt}\right)^2 + 18\frac{91\cos 2\theta - 9\sin^4\theta}{(9\sin^2\theta+91)^{3/2}}\left(\frac{d\theta}{dt}\right)^2$$

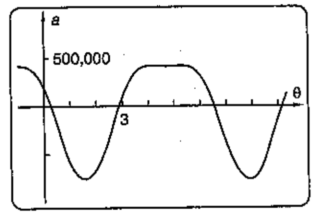
$$= \left[18\cdot\frac{91\cos 2\theta - 9\sin^4\theta}{(9\sin^2\theta+91)^{3/2}} - 6\sin\theta\right]\left(\frac{d\theta}{dt}\right)^2$$

(There are many other correct forms of the answer, depending on how you use the double-argument properties and Pythagorean properties from trigonometry.)

Note that the angular velocity is constant at 6000π radians per minute, so

$$\frac{d\theta}{dt} = 100\pi \text{ rad/s.}$$

d. See the graph. Note that a line at $a = -980$ is so close to the x-axis that it does not show up.



Solving graphically and numerically, $a < -980$ for $\theta \in (0.2712\ldots, 2.8703\ldots)$. The piston is going down ($v < 0$) for $\theta \in (\pi/2, 3\pi/2)$.

So the piston is going down with acceleration greater than gravity for θ between $\pi/2$ and $2.8703\ldots$.

Chapter Test

T1. $y = uv \Rightarrow y' = u'v + uv'$

T2. $y' = \lim_{\Delta x\to 0}\left[\dfrac{\dfrac{u+\Delta u}{v+\Delta v} - \dfrac{u}{v}}{\Delta x}\cdot\dfrac{(v+\Delta v)v}{(v+\Delta v)v}\right]$

$$= \lim_{\Delta x\to 0}\left[\frac{(u+\Delta u)v - u(v+\Delta v)}{\Delta x(v+\Delta v)v}\right]$$

$$= \lim_{\Delta x\to 0}\left[\frac{uv + \Delta uv - uv - u\Delta v}{\Delta x(v+\Delta v)v}\right]$$

$$= \lim_{\Delta x\to 0}\left[\frac{1}{(v+\Delta v)v}\cdot\frac{\Delta u\cdot v - u\Delta v}{\Delta x}\right]$$

$$= \lim_{\Delta x\to 0}\left[\frac{1}{(v+\Delta v)v}\cdot\left(\frac{\Delta u}{\Delta x}v - u\frac{\Delta v}{\Delta x}\right)\right]$$

$$= \frac{1}{v^2}\left(\frac{du}{dx}v - u\frac{dv}{dx}\right) = \frac{u'v - uv'}{v^2}$$

[Because as $\Delta x \to 0$, u/x and $\Delta v/\Delta x$ become du/dx and dv/dx and $v \to 0$, so $(v+\Delta v)v \to v^2$].

T3. $\cot x = \dfrac{\cos x}{\sin x}$

$$= \frac{-\sin x\sin x - \cos x\cos x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

T4. $y = \sin^{-1} x \Rightarrow \sin y = x \Rightarrow y'\cos y = 1 \Rightarrow$

$$y' = \frac{1}{\cos y}\cos^2 y + \sin^2 y = 1 \Rightarrow \cos^2 y = 1 - \sin^2 y \Rightarrow$$

$$\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2} \Rightarrow y = \frac{1}{\sqrt{1-x^2}}$$

T5. $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{4t^3}{2t} = 2t^2\ \dfrac{d^2y}{dx^2} = \dfrac{d}{dx}(2t^2) =$

$$4t(dt/dx) = 4t \div \frac{dx}{dt} = \frac{4t}{2t} = 2$$

T6. $c(x) = \cot 3x$

$c'(x) = -3 \csc^2 3x$, which is negative for all permissible values of x.

$c'(5) = -3 \csc^2 15 = -3/\sin^2 15 = -7.0943\ldots$

$c(t)$ is decreasing at about 7.1 y-units/x-unit.

T7. $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$

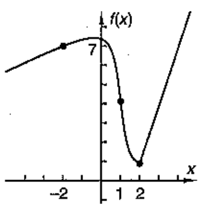
$f'(2) = \sec 2 \tan 2 = 5.25064633\ldots$

Use $m(x)$ for the difference quotient.

$$m(x) = \frac{1/\cos x - 1/\cos 2}{x - 2}$$

x	$m(x)$
1.997	5.28893631...
1.998	5.27611340...
1.999	5.26335022...
2.000	undefined
2.001	5.23800134...
2.002	5.22541482...
2.003	5.21288638...

T8. Answers may vary.



T9. $f(x) = mx + b$

$f'(x) = m$ for all x

$\therefore f$ is differentiable for all x.

$\therefore f$ is continuous for all x, Q.E.D.

T10. $f(x) = \sec 5x \Rightarrow f'(x) = 5 \sec 5x \tan 5x$

T11. $y = \tan^{7/3} x \Rightarrow y' = \frac{7}{3} \tan^{4/3} x$

T12. $f(x) = (2x - 5)^6(5x - 1)^2$

$$f'(x) = 6(2x - 5)^5(2) \cdot (5x - 1)^2 + (2x - 5)^6 \cdot 2(5x - 1) \cdot 5$$
$$= 2(2x - 5)^5(5x - 1)[6(5x - 1) + 5(2x - 5)]$$
$$= 2(2x - 5)^5(5x - 1)(40x - 31)$$

T13. $f(x) = \dfrac{e^{3x}}{\ln x} \Rightarrow$

$$f'(x) = \frac{3e^{3x} \ln x - e^{3x}(1/x)}{(\ln x)^2} = \frac{3xe^{3x} \ln x - e^{3x}}{x(\ln x)^2}$$

T14. $x = \sec 2t$

$y = \tan 2t^3$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sec^2 2t^3 \cdot 6t^2}{\sec 2t \tan 2t \cdot 2} = \frac{3t^2 \sec^2 2t^3}{\sec 2t \tan 2t}$$

T15. $y = 4 \sin^{-1}(5x^3)$

$$y' = 4 \cdot \frac{1}{\sqrt{1-(5x^3)^2}} \cdot 15x^2 = \frac{60x^2}{\sqrt{1-25x^6}}$$

T16. $9x^2 - 20xy + 25y^2 - 16x + 10y - 50 = 0 \Rightarrow$

$18x - 20y - 20xy' + 50yy' - 16 + 10y' = 0$

$y'(-20x + 50y + 10) = -18x + 20y + 16$

$$y' = \frac{dy}{dx} = \frac{-18x + 20y + 16}{-20x + 50y + 10}$$
$$= \frac{-9x + 10y + 8}{-10x + 25y + 5}$$

If $x = -2$, then

$36 + 40y + 25y^2 + 32 + 10y - 50 = 0$

$25y^2 + 50y + 18 = 0$

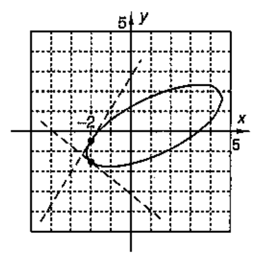
Solving numerically gives $y = -0.4708\ldots$ or $y = -1.5291\ldots$, both of which agree with the graph.

(Solving algebraically by the quadratic formula, $y = -1 \pm \sqrt{7}/5$, which agrees with the numerical solutions.)

At $(-2, -0.4708\ldots)$, $dy/dx = 1.60948\ldots$.

At $(-2, -1.5291\ldots)$, $dy/dx = -0.80948\ldots$.

The answers are reasonable, because lines of these slopes are tangent to the graph at the respective points, as shown here.



T17. $f(x) = \begin{cases} x^3 + 1, & \text{if } x \le 1 \\ a(x-2)^2 + b, & \text{if } x > 1 \end{cases}$

$f'(x) = \begin{cases} 3x^2, & \text{if } x < 1 \\ 2a(x-2), & \text{if } x > 1 \end{cases}$

For equal derivatives on both sides of $x = 1$,

$\lim_{x \to 1^-} f'(x) = 3 \cdot 1^2 = 3$

$\lim_{x \to 1^+} f'(x) = 2a(1-2) = -2a$

$\therefore -2a = 3 \Rightarrow a = -1.5$

For continuity at $x = 1$,

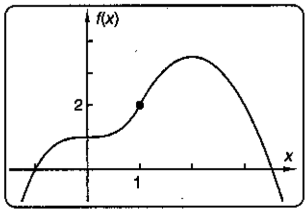
$\lim_{x \to 1^-} f(x) = 1^3 + 1 = 2$

$\lim_{x \to 1^+} f(x) = a(1-2)^2 + b = a + b$

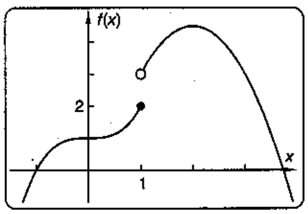
$\therefore a + b = 2$

Substituting $a = -1.5$ gives $b = 3.5$.

The graph shows differentiability at $x = 1$.



Values of b other than 3.5 will still cause the two branches to have slopes approaching 4 as x approaches 1 from either side as long as $a = -1.5$. However, f will not be continuous, and thus will not be differentiable, as shown here for $b = 4.5$.



T18. $y = x^{7/3} \Leftrightarrow y^3 = x^7$

$3y^2y' = 7x^6$

$y' = \dfrac{7x^6}{3y^2} = \dfrac{7}{3}\dfrac{x^6}{(x^{7/3})^2} = \dfrac{7}{3}x^{6-14/3} = \dfrac{7}{3}x^{4/3}$

This answer agrees with $y' = nx^{n-1}$. 4/3 is 7/3 − 1.

T19. cot = adjacent/opposite = $x/5$ = $\cot^{-1}(x/5)$

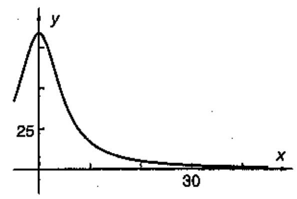
T20. $\dfrac{d\theta}{dx} = -\dfrac{1}{1+(x/5)^2} \cdot \dfrac{1}{5} = -\dfrac{1}{5+5(x^2/25)}$

$= -\dfrac{1}{5+(x^2/5)} = -\dfrac{5}{25+x^2}$

T21. $\dfrac{dx}{dt} = -420$ mi/h

T22. $\dfrac{d\theta}{dt} = \dfrac{d\theta}{dx} \cdot \dfrac{dx}{dt} = -\dfrac{5}{25+x^2} \cdot (-420) = \dfrac{2100}{25+x^2}$

T23. The plane is changing fastest when x approaches zero, when the plane is nearest the station.



T24. Answers will vary.