Name: ______________ Group Members: ______________

Exploration 2-1a: Introduction to Limits

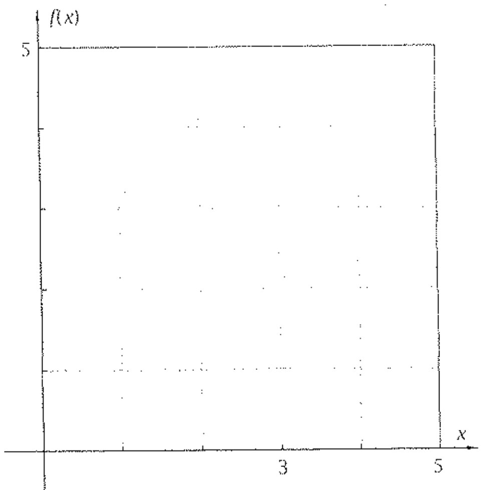
Date: ______________

Objective: Find the limit of a function that approaches an indeterminate form at a particular value of x and relate it to the definition.

1. Plot on your grapher the graph of this function.

$$f(x) = \frac{x^3 - 7x^2 + 17x - 15}{x - 3}$$

Use a friendly window with $x = 3$ as a grid point, but with the grid turned off. Sketch the results here. Show the behavior of the function in a neighborhood of $x = 3$.



2. Substitute 3 for x in the equation for $f(x)$. What form does the answer take? What name is given to an expression of this form?

3. The graph of f has a **removable discontinuity** at $x = 3$. The y-value at this discontinuity is the **limit** of $f(x)$ as x approaches 3. What number does this limit equal?

4. Make a table of values of $f(x)$ for each 0.1 unit change in x-value from 2.5 through 3.5.

x	$f(x)$
2.5	
2.6	
2.7	
2.8	
2.9	
3.0	
3.1	
3.2	
3.3	
3.4	
3.5	

5. Between what two numbers does $f(x)$ stay when x is kept in the open interval (2.5, 3.5)?

6. Simplify the fraction for $f(x)$. Solve numerically to find the two numbers close to 3 between which x must be kept if $f(x)$ is to stay between 1.99 and 2.01.

long division

$x-3 \overline{) x^3 - 7x^2 + 17x - 15}$

$$f(x) = \frac{(x-3)(\quad)}{x-3}$$

7. How far from $x = 3$ (to the left and to the right) are the two x-values in Problem 6?

8. For the statement "If x is within ______ units of 3 (but not equal to 3), then $f(x)$ is within 0.01 unit of 2," write the largest number that can go in the blank.

9. The formal definition of limit is

$$L = \lim_{x \to c} f(x) \text{ if and only if}$$

- for any positive number ε (no matter how small)
- there is a positive number δ such that
- if x is within δ units of c, but not equal to c,
- then $f(x)$ is within ε units of L.

The four numbers L, c, ε, and δ all appear in Problem 8. Which is which?

10. What did you learn as a result of doing this Exploration that you did not know before?