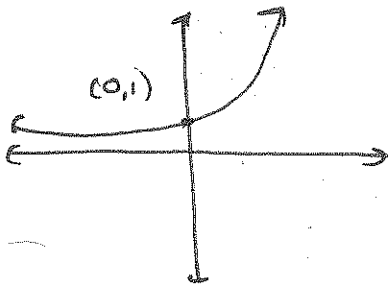
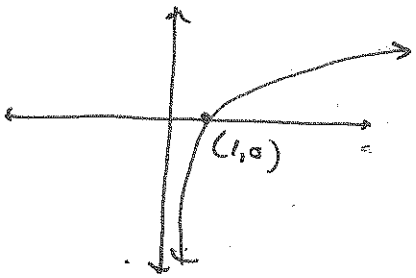
Ch. 9 Review of Logarithms

1. Sketch & label 1 point: $f(x) = e^x$



$g(x) = \ln x$



2. What is the relationship between the graphs? inverses

State whether each sentence is true for $f(x)$, $g(x)$, both or neither:

3. The function is strictly increasing. f(x) & g(x)
4. The function passes through (0,1). f(x)
5. The domain includes the positive real numbers. f(x) + g(x)
6. The function has an asymptote at $x = 0$. g(x)
7. The range includes all real numbers. g(x)

Evaluate without a calculator:

8. $\log_4 64$ 3
9. $\log_2 16$ 4
10. $\log_5 125$ 3
11. $\log 10000$ 4
12. $\log 1$ 0
13. $\ln e^2$ 2
14. $\ln 1$ 0
15. $64^{\frac{2}{3}}$ 16
16. $36^{-\frac{1}{2}}$ $\frac{1}{6}$

Simplify using properties, then evaluate:

17. $\log 20 + \log 5$ $\log 100 \rightarrow 2$
18. $\log 1000^4$ $4 \log 1000$ $4 \cdot 3 = 12$
19. $\log 3500 - \log 35$ $\log 100 \rightarrow 2$

Write as a single logarithm:

20. $\log 5 + \log 3$ $\log 15$
21. $\log 8 - \log 2$ $\log 4$
22. $3\log x + \log 17$ $\log 17x^3$
23. $3\log c - \log d$ $\log\left(\frac{c^3}{d}\right)$
24. $\frac{1}{2}\log A + \log B$ $\log(A^{\frac{1}{2}}B)$

Simplify. NO CALCULATOR! Work smarter, not harder!

25. $10^{\log 5}$ 5
26. $10^{\log 2}$ 2
27. $10^{\log 3.7}$ 3.7
28. $e^{\ln 5}$ 5
29. $e^{\ln 4.7}$ 4.7
30. $e^{\ln 2.3}$ 2.3
31. $\ln e^3$ 3
32. $\log_5 5^3$ 3

Solve, showing work algebraically (symbolically). Use a calculator.

33. $2^t = 3$

$t \log 2 = \log 3$

$t = \log 3 / \log 2$

$= 1.5850$

34. $\log_{17} 34$

$\frac{\log 34}{\log 17}$

$= 1.2447$

35. $\log_4 128$

$\frac{\log 128}{\log 4} = 3.5$

36. $\ln 3x = \ln(2x+17)$

$3x = 2x + 17$

$x = 17$

37. $5^x = 32$

$x \ln 5 = \ln 32$

$x = \ln 32 / \ln 5 = 2.1534$

38. Write without logs: $\log x = 3\log B + 4\log C$

$\log x = \log(B^3 \cdot C^4)$

$x = B^3 \cdot C^4$

Answer the following on a separate sheet of paper, showing work.

39. The population in the U.S. in 1990 was 250 million, with continuous growth of 1%. When will the population be 300 million? $300 = 250 e^{0.01t}$

$t = 18.2322$

$1990 + 19 = \boxed{2009}$

40. The half-life of C-14 is 5730 years. If 63% of the original C-14 remains in a skull, approximately how old is the skull?

$.5 = b^{5730}$

$b = 0.999879$

$0.63 = (0.999879)^t$

$t = \ln 0.63 / \ln(0.999879) = 3818.2439$

41. Write the EQUATION only for each situation:

a) daily compounding at 6% for 3 years $A = P\left(1 + \frac{0.06}{365}\right)^{365(3)}$

b) continuous compounding at 6% for 3 yrs $A = P \cdot e^{0.06(3)}$

42. An estimate for Earth's population P in billions is $P = 4(2)^{\frac{y-1975}{35}}$, where y is the year. (a) Evaluate P for $y = 1975$. (b) Estimate P in 2000. (c) When will the 1990 population be doubled?

(a) $P = 4$ (b) $P \approx 6.562$ billion (c) 2025

43. The half-life of a certain substance is 30 hours. About how many hours will it take for an initial amount of 8 grams to decay to 2.5 grams? ≈ 50 hours

44. The intensity of sunlight at points below the ocean is thought to decrease exponentially with the depth of the water. One model for this situation is $I = 100(0.325)^d$. If special equipment is needed for divers to see when the intensity drops below 0.2 units, at what depth (d) is this equipment needed? $d = 5.5294$ m

45. The population of Tanzania in 1995 was about 28.7 million with an annual growth rate of 2.6%. Assume the population changes continuously. (a) Give an equation for the population n years after 1995. (b) Predict the population in 2000. (c) Predict when the population will reach 35 million.

(a) $P = 28.7 e^{.026n}$ (b) 32.6844 million (c) 2003

46. A certain radioactive substance has a half-life of 4 hours. Let A be the original amount of the substance and L the amount left after h hours. (a) Give an exponential model for L in terms of A and h. (b) What percent of the original amount of the substance will remain after 7 hours?

(a) $L = A(0.8409)^h$ (b) $\approx 29.7302\%$

47. The formula $\log w = -2.866 + 2.722 \log h$ estimates the normal weight w of a girl h inches tall. Estimate the normal weight of a girl who is 55 inches tall.

$w = 74.3477$ lbs.