Solving Exponential Equations

Example

Solve $125 = 25^x$.

1. Get same bases. $125 = 5^3$ and $25 = 5^2$.
2. Set exponents equal to each other.
3. Solve.

$$5^3 = (5^2)^x$$
$$3 = 2x$$
$$\frac{3}{2} = x$$

Solve.

1. $2^x = 8$

2. $9^x = 27$

3. $4^x = \dfrac{1}{16}$

4. $4^{3t} = 64$

5. $9^{4y} = \dfrac{1}{81}$

6. $3^{3x-1} = 9^{x-1}$

7. $5^{2x-2} = 25^{2x+5}$

8. $9^{2-x} = 27^{x/3}$

Converting Logarithmic and Exponential Forms

$y = \log_b x$ is equivalent to $x = b^y$.

Example

Write $6^x = 216$ in logarithmic form. $x = \log_6 216$

Write $\log_5 625 = x$ in exponential form. $5^x = 625$

Write problems 1–5 in logarithmic form. Write problems 6–10 in exponential form.

1. $3^4 = 81$

2. $10^{-1} = 0.1$

3. $8^{-1/3} = \dfrac{1}{2}$

4. $10^4 = 10000$

5. $15^{5/2} = 225\sqrt{15}$

6. $\log_2 8 = 3$

7. $\log_5 125 = 3$

8. $\log_3 \dfrac{1}{27} = -3$

9. $\log_{3.2} 3.2 = 1$

10. $\log_{2/3} \dfrac{27}{8} = -3$

Solving Logarithmic Equations

$$\log_4 \frac{1}{4} = \frac{x}{64}$$

1. Rewrite in exponential notation.

$$4^{x/64} = \frac{1}{4}$$

2. Write with common base.

$$4^{x/64} = 4^{-1}$$

3. Set exponents equal to each other.

$$\frac{x}{64} = -1$$

4. Solve.

$$x = -64$$

Solve.

1. $\log_x 25 = 2$

2. $\log_{10} x = 3$

3. $\log_x 3 = \frac{1}{2}$

4. $\log_3 27 = 3x + 6$

5. $\log_7 \frac{1}{49} = -x - 4$

6. $\log_{1/10} 100 = x$

7. $\log_{2/3} \frac{4}{9} = 2x$

8. $\log_{1/3} 9 = x + 3$

Evaluating Common and Natural-Logarithms

If $\log A = y$ then $\log_{10} A = y$ and $10^y = A$.

If $\ln x = y$ then $\log_e x = y$ and $e^y = x$.

Use the 10^x and e^x keys found on a scientific calculator to find the following.

$\log x = 0.02$	$\ln x = 5.2$
1. Rewrite. $10^{0.02} = x$	1. Rewrite. $e^{5.2} = x$
2. Solve. $1.047 \doteq x$	2. Solve. $181.272 \doteq x$

Use a calculator to solve for x. Round answers to the nearest thousandth.

1. $\log x = 3.963$

2. $\log x = 0.0238$

3. $\log x = -0.3421$

4. $\log x = -0.98$

5. $\ln x = 1.304$

6. $\ln x = 8.628$

7. $\ln x = 0.0832$

8. $\ln x = -5.2$

Common and Natural Logarithms

A common logarithm is a log with base = 10.

$\log A = \log_{10} A$

A natural logarithm (ln) is a log with base e.

(e is used in many applications.)

$\ln x = \log_e x$

Both log and ln keys can be found on a scientific calculator.

Find log 0.08 using a calculator.   Find ln 5.2 using a calculator.

$\log 0.05 \approx$ **-1.097**   ln 5.2 $\approx$ **1.649**

Solve. Round answers to the nearest thousandth.

1. log 3.7

2. log 322

3. log 0.05

4. log 2 + log 6

5. $\dfrac{\log 151}{\log 9}$

6. ln 4.5

7. ln 62

8. ln 640

9. ln 1000 ÷ ln 500

10. $\dfrac{\ln 24}{\ln 2}$

Rational Exponents

$\sqrt[n]{a} = a^{1/n}$ and $\sqrt[n]{a^m} = a^{m/n}$

$\sqrt[4]{(2a)^3} = (2a)^{3/4}$ $\left(\dfrac{2r}{7s}\right)^{-5/2} = \left(\dfrac{7s}{2r}\right)^{5/2}$ $(a^{-1/2} b^{3/4})^{1/3} = a^{(-1/2)(1/3)} b^{(3/4)(1/}$

$= a^{-1/6} b^{1/4}$

$= \dfrac{b^{1/4}}{a^{1/6}}$

Simplify. Rewrite with positive, rational exponents.

1. $\sqrt[3]{x^2}$

2. $\sqrt[5]{xy^2z^3}$

3. $\left(\dfrac{3a}{4b}\right)^{-3/5}$

4. $(\sqrt[8]{6^3})^{2/7}$

5. $2^{1/4} \cdot 2^{2/5}$

6. $x^{1/7} \cdot x^{3/7} \cdot x^{2/7}$

7. $g^{2/3} \cdot g^{5/4}$

8. $(x^{-1/3} y^{-2/5})^{-15}$

9. $\dfrac{c^{-4/5} d^{5/9}}{c^{7/10} d^{1/6}}$

10. $\sqrt[3]{\sqrt{xy}}$

Answer Pages

Page 40
1. $5x^3 - x^2 - 10x + 2$ 2. $x^2 + 3x - 2$
3. $3x^3 - 6x$ 4. $-x^2 + 5x + 1$
5. 98   6. -63

Page 41
1. Function and 1-1
2. Function, not 1-1
3. Function and 1-1
4. Neither function nor 1-1

Page 42
1. $f^{-1}(x) = \frac{1}{4}x + 2$ 2. $f^{-1}(x) = \frac{1}{3}x - 2$

3. $f^{-1}(x) = 2x + 2$ 4. $f^{-1}(x) = \frac{3}{2}x - 6$

5. $f^{-1}(x) = \frac{4}{5(x+3)}$ 6. $f^{-1}(x) = \sqrt[3]{x}$

Page 43
1. 3   2. $\frac{3}{2}$ 3. -2   4. 1
5. $-\frac{1}{2}$ 6. -1   7. -6   8. $\frac{4}{3}$

Page 44
1. $\log_3 81 = 4$ 2. $\log_{10} 0.1 = -1$
3. $\log_8 \frac{1}{2} = -\frac{1}{3}$ 4. $\log_{10} 10000 = 4$
5. $\log_{15} 225\sqrt{15} = \frac{5}{2}$ 6. $2^3 = 8$
7. $5^3 = 125$ 8. $3^{-3} = \frac{1}{27}$
9. $3.2^1 = 3.2$ 10. $(\frac{2}{3})^{-3} = \frac{27}{8}$

Page 45
1. 5   2. 1000   3. 9   4. -1
5. -2   6. -2   7. 1   8. -5

Page 46
1. 0.568   2. 2.508   3. -1.301   4. 1.079
5. 2.283   6. 1.504   7. 4.127   8. 6.461
9. 13.122   10. 4.585

Page 47
1. 9183.326   2. 1.056
3. 0.455   4. 0.105
5. 3.684   6. 5585.895
7. 1.087   8. 0.006

Page 48
1. 0.944   2. 2   3. 2.814   4. -1.135
5. 1   6. 0.778   7. -5.672   8. 0.803
9. 2.096   10. 5.904

Page 49
1. 1.34   2. 9.83   3. 1.93   4. 0.64
5. -0.27   6. 12.02

Page 50
1. $\log_4 3 + \log_4 y$ 2. $\log_a b - \log_a c$

3. $\frac{1}{2}\log_4 5$ 4. $\log_5 2 + 3\log_5 x$

5. $2y(\log_b x - \log_b 5)$ 6. $\log_4(xyz)$

7. $\log_2 m^5 n$ 8. $\log_3(\frac{\sqrt{x}}{y})$

9. $\log_3(\frac{a}{b^{1/4}})$ 10. $\log_4(\frac{xy}{z})$

Page 51
1. $\sqrt{3}$ 2. 8   3. 1   4. $\sqrt{6}$
5. 7, -7   6. $-\frac{4}{3}$

Page 28
1. $x^{2/3}$ 2. $x^{1/5}y^{2/5}z^{3/5}$ or $(xy^2z^3)^{1/5}$

3. $(\frac{4b}{3a})^{3/5}$ 4. $6^{3/28}$

5. $2^{13/20}$ 6. $x^{6/7}$

7. $g^{23/12}$ 8. x^5y^6

9. $\frac{d^{7/18}}{c^{3/2}}$ 10. $(xy)^{1/6}$ or $x^{1/6}y^{1/6}$