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Chapter 13

Polar Coordinates

Lesson 13-3 (pp. 797-803)

Mental Math

$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Guided Example 2

2, $\frac{11\pi}{4}$; $\frac{7\pi}{4}$; –2, $-\frac{\pi}{4}$; –2, $\frac{7\pi}{4}$

Activity

Step 1:

a. [4.5, 153°]

b. [6.1, 189°]

c. [2.0, 270°]

d. [5.8, 329°]

Step 2:

e. (1.6, 2.5)

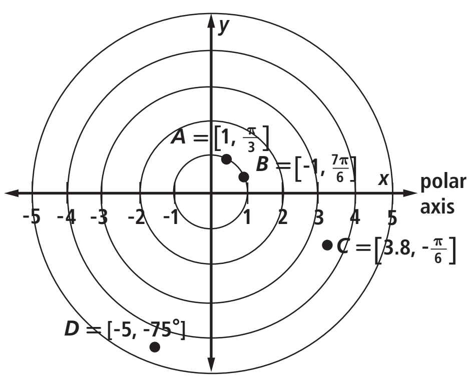
f. (–4.0, 0.0)

g. (1.5, 2.6)

h. (–0.7, –0.7)

Questions

1. a. – d.



2. a. $\left[7, \frac{17\pi}{6}\right]$

   b. $\left[-7, \frac{11\pi}{6}\right]$

3. (0, 5)

4. (4.24, –4.24)

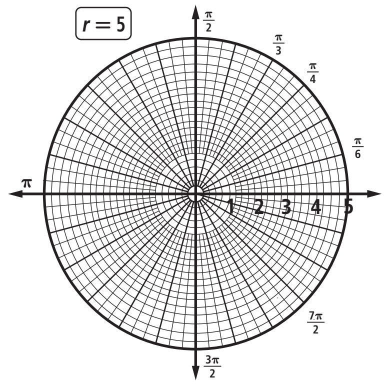
5. (–3.46, –2)

6. $\left[30, \frac{\pi}{3}\right]$

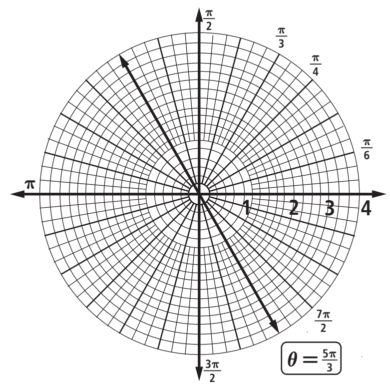
7. [10, –53.1°]

8. C

9.



10.



11. true

12. false; $|r| = \sqrt{x^2 + y^2}$

13. $\theta = \frac{11\pi}{4}$; It is a line extending from the origin through Quadrants II and IV.

14. $r = 12$; It is a circle with radius 12.

15. $\left[-8, \frac{2\pi}{3}\right]$

16. $\left[8, \frac{\pi}{3}\right]$

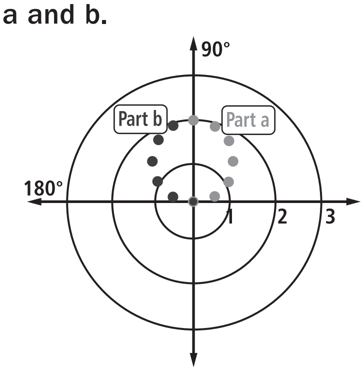
17. a. 30° west of south

    b. 10° west of north

    c. 20° south of west

    d. 50° west of north

18. a and b.



    c. The graph of all points satisfying this equation is a circle with radius 1 and center [1, 90°].

19. The equation is an identity. $\sin\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2}\right)\cos(x) + \cos\left(\frac{\pi}{2}\right)\sin(x) = 1 \cdot \cos x + 0 \cdot \sin x = \cos x$; domain: all real numbers.

20. The equation is not an identity. Let $x = \frac{\pi}{3}$.
Then $\sec\left(\frac{\pi}{3}\right) + \cot\left(\frac{\pi}{3}\right)\csc\left(\frac{\pi}{3}\right) = \frac{8}{3}$ and
$\sec\left(\frac{\pi}{3}\right)\csc\left(\frac{\pi}{3}\right) = \frac{4\sqrt{3}}{3} \neq \frac{8}{3}$.

21. a. 1
b. 36

22. a. $\sec^2 \theta$
b. $r^2\sec^2 \theta$

23. true

24. Answers vary. Sample: Name the vertices *A*, *B*, *C*, *D*, respectively. For the quadrilateral to be a parallelogram, $\overline{AB}$ must be parallel to $\overline{DC}$, and $\overline{BC}$ must be parallel to $\overline{AD}$, so we must show that these segments have equal slopes. $m_{\overline{AB}} = \frac{n-0}{m-0} = \frac{n}{m}$ and $m_{\overline{DC}} = \frac{q-(q-n)}{p-(p-m)} = \frac{n}{m}$; $m_{\overline{BC}} = \frac{q-n}{p-m}$ and $m_{\overline{AD}} = \frac{(q-n)-0}{(p-m)-0} = \frac{q-n}{p-m}$. Since both pairs of opposite sides are parallel, the figure is a parallelogram

25. Answers vary. Sample: Polar coordinates are used in two different systems that deal with three-dimensional space. With cylindrical coordinates, a point (*x*, *y*, *z*), where *x* and *y* are the typical Cartesian axes and *z* is a vertical axis perpendicular to the plane, is described by [*r*, θ, *z*] in cylindrical coordinates. Here, as in polar coordinates, $x = r \cos \theta$ and $y = r \sin \theta$, and therefore the *z*-axis is still perpendicular to the plane. In spherical coordinates, there is a single origin point, which we can visualize as the center of a sphere. Once again the polar system is laid onto the Cartesian system, with the *z*-axis perpendicular to the plane. But now the polar system is described by [*r*, θ, ϕ], where θ is the angle between the *x*-axis and a ray extending to a point on the sphere, and ϕ is the angle between the *z*-axis and said ray.