Chapter 13

The Geometry of Complex Numbers

Lesson 13-5 (pp. 811-816)

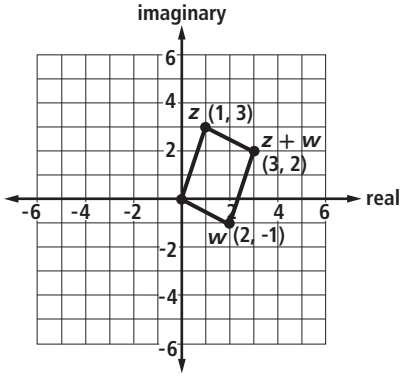
Mental Math

a. $\begin{bmatrix} -1 & 8 \\ 6 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & -4 \\ 0 & 9 \end{bmatrix}$

Activity

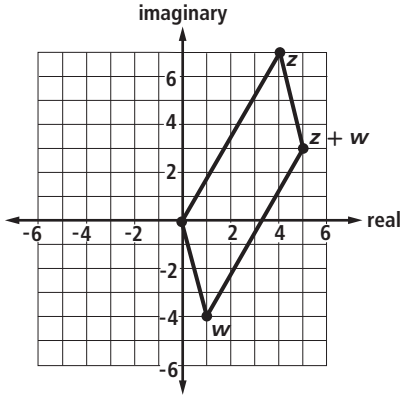
Steps 1 and 2:



Step 1: Answers vary. Sample:
$z = (a, b) = (1, 3)$; $w = (c, d) = (2, -1)$

Step 2: Answers vary. Sample: $z + w = 3 + 2i$

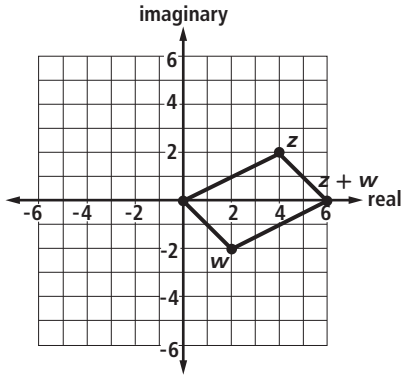
Step 3: Answers vary. Sample:



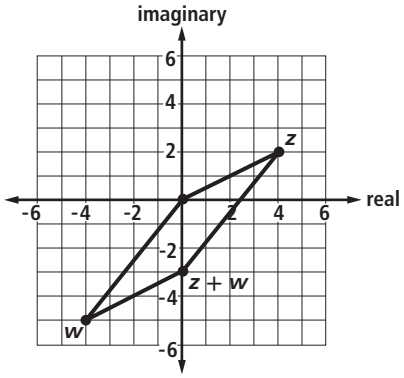
$z + w$ is a parallelogram.

Step 4:

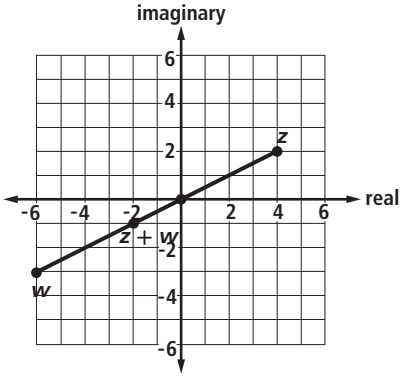
a. $w = c - 2i$, where c is any real number.



b. $w = -4 + di$, where d is any real number.



c. $w = c + di$, where $c = 2d$



Guided Example 2

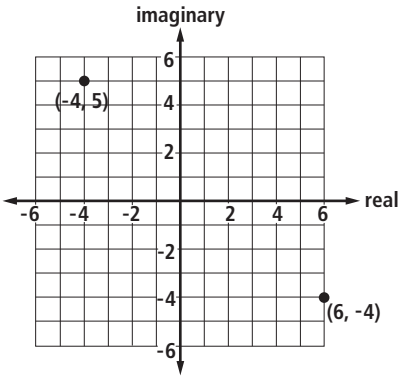
a. $-1 + 0i$; $-2 + 3i$

b. $y + 1$; 1

Questions

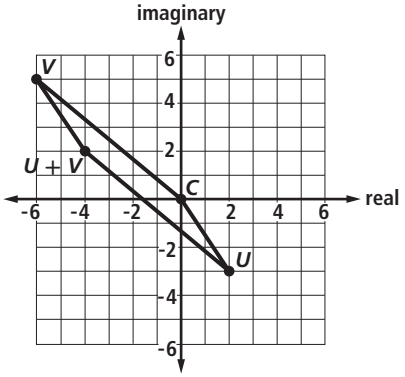
1. real; imaginary
2. a. (6, –4)
3. a. (–4, 5)

2b and 3b

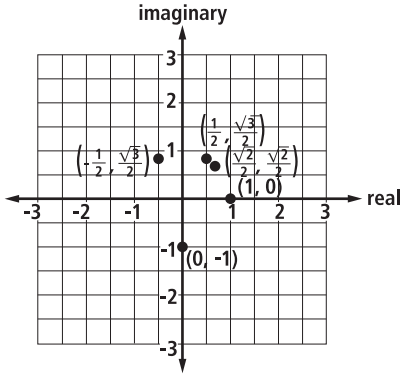


4. $A = 2.1 + 3i$, $B = -3 + 4i$, $C = -4 - 4i$, $D = -6i$.

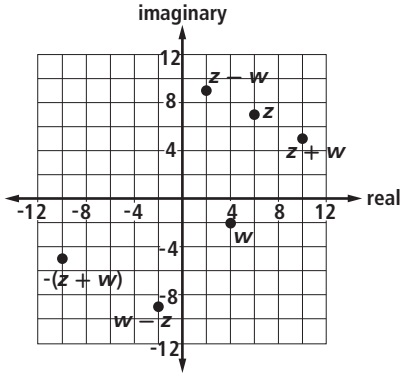
5. a. $U + V = -4 + 2i$
   b and c.



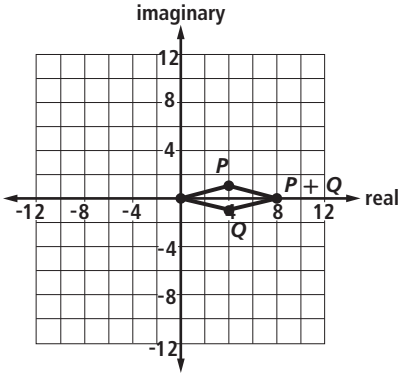
6. Graph the quadrilateral *ABCD* on the real-imaginary plane, where $A = (0, 0)$, $B = (a, b)$, $C = (a + c, b + d)$, and $D = (c, d)$. The figure is a parallelogram if and only if the slope of $\overline{AB}$ = the slope of $\overline{DC}$, and the slope of $\overline{AD}$ = the slope of $\overline{BC}$. Slope of $\overline{AB} = \frac{b-0}{a-0} = \frac{b}{a}$, and slope of $\overline{DC} = \frac{(b+d)-d}{(a+c)-c} = \frac{b}{a}$, so $\overline{AB}$ and $\overline{DC}$ are parallel. Slope of $\overline{AD} = \frac{d-0}{c-0} = \frac{d}{c}$ and slope of $\overline{BC} = \frac{(b+d)-b}{(a+c)-a} = \frac{d}{c}$, so $\overline{AD}$ and $\overline{BC}$ are parallel. Thus the quadrilateral is a parallelogram.
7. [5, 5.3559]
8. $\left[1, -\frac{\pi}{3}\right]$
9. $|a + bi| = \sqrt{a^2 + b^2}$
10. a. 13
    b. 90°
11. a. $3\sqrt{5}$
    b. 153.4°
12. Answers vary. Sample:
    $\frac{1}{2} + \frac{\sqrt{3}}{2}i, -i, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, 1$



13. a. i. – iv.

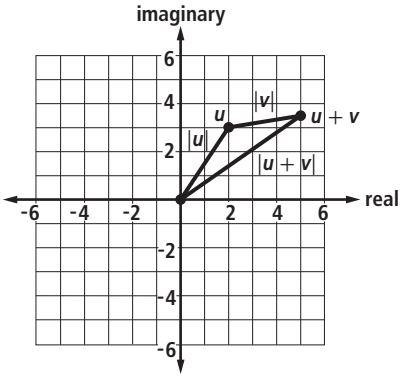


    b. Let $A = z - w = (2, 9)$, $B = z + w = (10, 5)$, $C = w - z = (-2, -9)$, and $D = -(z + w) = (-10, -5)$. Slope of $\overline{AB} = -\frac{1}{2}$ and slope of $\overline{DC} = -\frac{1}{2}$, so $\overline{AB}$ and $\overline{DC}$ are parallel. Slope of $\overline{AD} = \frac{7}{6}$ and slope of $\overline{BC} = \frac{7}{6}$, so $\overline{AD}$ and $\overline{BC}$ are parallel. Thus the quadrilateral *ABCD* is a parallelogram.
14. a.

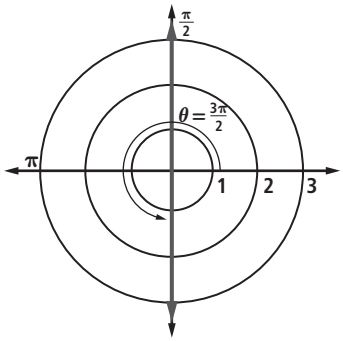


    b. rhombus
    c. 8
15. Yes; the distance from u to v is $\sqrt{(a-c)^2 + (b-d)^2}$ and $|u - v| = |(a - c) + i(b - d)| = \sqrt{(a-c)^2 + (b-d)^2}$.
16. a. 16
    b. $4\sqrt{5}$

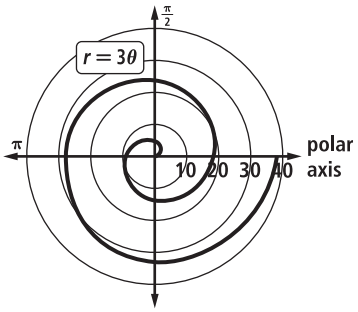
c. The triangle inequality states that the sum of any two sides of a triangle is longer than the third side; the sum of the two sides is equal to the third side only if the three points are on the same line. Therefore, if two sides are $|u|$ and $|v|$, the third side has measure $|u + v|$, then consequently $|u| + |v| \leq |u + v|$.



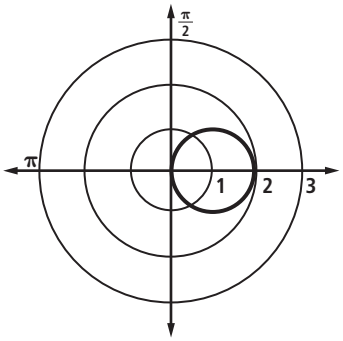
17.



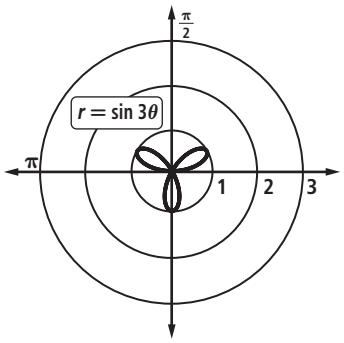
18.



19.



20.



21. a. The relation is not a function.
    b. yes

22. (a), (b), and (c)

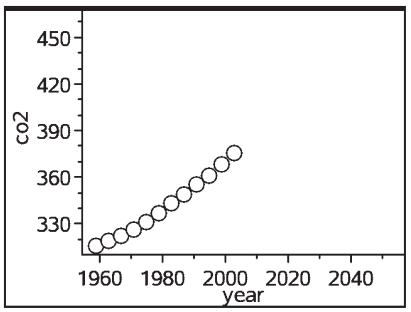
23. divergent

24. convergent, 25

25. convergent, $\frac{64}{7}$

26. a. linear: 377.8 ppmv, 437.06 ppmv; exponential: 379.01 ppmv, 450.46 ppmv

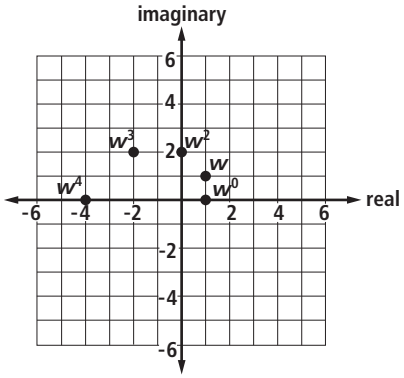
    b.



    c. The fact that the annual increase in average atmospheric CO_2 does not increase at a constant rate over time indicates that an exponential model might be better suited to this situation than a linear model.

27. a. 1, $2i$, $-2 + 2i$, -4

    b.



    c. The graph forms a spiral about the origin as the coordinates continue to expand outwards.