3-7 Lesson Master

Questions on SPUR Objectives
See Student Edition pages 216–219 for objectives.

SKILLS Objective A

In 1 and 2, let $f(x) = x^2 + 7x + 2$ and $g(x) = 3x - 5$.

1. Evaluate each composite.

   a. $f(g(1))$ ______________ b. $g(f(1))$ ______________

2. Find a formula for each composite.

   a. $f(g(x))$ ______________ b. $g(g(x))$ ______________

3. Let $F = \{(2, 8), (3, 5), (4, 3), (5, 2)\}$ and $G = \{(8, 2), (2, 4), (3, 3), (5, 2)\}$. Find each composite.

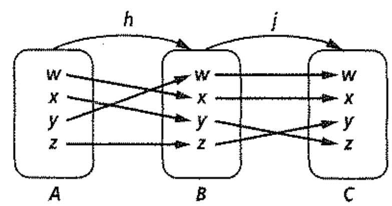
   a. $F \circ G$ ______________ b. $G \circ F$ ______________

4. Consider the functions h mapping A to B and j mapping B to C. Evaluate each composite.

   a. $h(j(w))$ ______________

   b. $j(h(x))$ ______________

   c. $(h \circ j)(z)$ ______________



5. Let $S: (x, y) \rightarrow (x, 2y)$ and let $T: (x - 3, y + 4)$. Write a simplified formula for $(T \circ S)(x, y)$. ______________

PROPERTIES Objective F

6. Let $s(x) = \sqrt{x - 2}$ and $n(x) = x^2 - 1$. Give the domain of each composite.

   a. $n \circ s$ ______________ b. $s \circ n$ ______________

In 7 and 8, true or false. If true, justify your answer. If false, give a counterexample.

7. Let $g(t) = \frac{1}{t} - 1$. The domain of g is the same as the domain of $g \circ g$. ______________

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8. A horizontal translation of h units followed by a ~~vertical~~ horizontal scale change of magnitude p is the same as a ~~vertical~~ scale change of magnitude p followed by a horizontal translation of h units. horizontal

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3-8 Lesson Master

Questions on SPUR Objectives
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SKILLS Objective B

In 1–4, a function is described. a. Give a set of ordered pairs or an equation for the inverse of the function; b. State whether the inverse is a function.

1. $y = 2 - 3x$ a. ____________ b. ____________

2. $f(x) = \frac{1}{\sqrt{x}}$ a. ____________ b. ____________

3. $g = \{(3, 2), (2, -1), (5, 3), (3, -1)\}$

 a. ____________ b. ____________

4. $h(x) = \frac{3}{x + 4}$ a. ____________ b. ____________

PROPERTIES Objective F

In 5 and 6, true or false. If true, explain your answer. If false, give a counterexample.

5. If a function is an even function, then its inverse is not a function.

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6. Given two functions, f and g, if $f(g(x)) = x$ for all x in the domain of g then f and g are inverses. ____________

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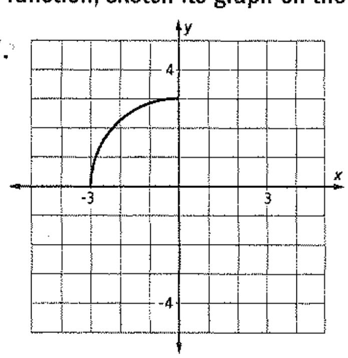
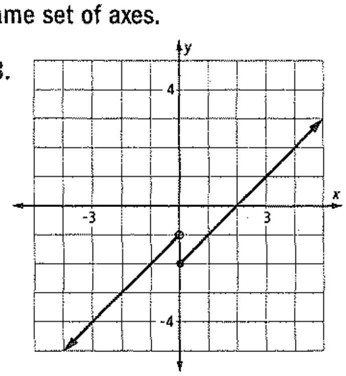
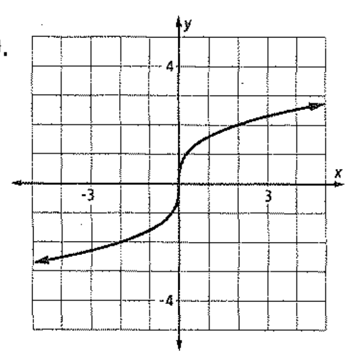
REPRESENTATIONS Objectives I, K

Miras!

In 7–9, determine whether the inverse of the graphed function is a function. If the inverse is a function, sketch its graph on the same set of axes.

7. 

8. 

9. 

____________ ____________ ____________

Composite Functions

Example:

If $f(x) = x + 3$ and $g(x) = 2x^2$

Find $f(g(x))$.

$$f(g(x)) = f(2x^2) = \mathbf{2x^2 + 3}$$

Find $g(f(x))$.

$$\begin{aligned} g(f(x)) &= g(x + 3) \\ &= 2(x + 3)^2 \\ &= 2(x^2 + 6x + 9) \\ &= \mathbf{2x^2 + 12x + 18} \end{aligned}$$

$f(x) = 3x - 5$ $\quad$ $g(x) = x^2 - 1$ $\quad$ $h(x) = x + 3$

ind the composition of the following functions.

1. $f(g(x))$

2. $g(h(x))$

3. $h(f(x))$

4. $g(f(x))$

5. $f(h(x))$

6. $f(h(g(x)))$

Inverse Functions

A function f has an inverse only if the function f is one-to-one. (I passes the vertical and horizontal line tests.)
The inverse function is denoted as f^{-1}.

Example:

Find the inverse of $f(x) = 2x + 8$.

1. Replace $f(x)$ with y. $\quad y = 2x + 8$

2. Interchange x and y. $\quad x = 2y + 8$

3. Solve for y. $\quad x - 8 = 2y$

$$\frac{x}{2} - 4 = y$$

$$\mathbf{f^{-1}(x) = \frac{x}{2} - 4}$$

Find the inverse of the following functions.

1. $f(x) = 4x - 8$

2. $f(x) = 3x + 6$

3. $f(x) = \frac{x}{2} - 1$

4. $f(x) = (\frac{2}{3})x + 4$

5. $f(x) = \frac{4}{5x} - 3$

6. $f(x) = x^3$

Back to Lesson 3-7

Name

3-7 Lesson Master

Questions on SPUR Objectives
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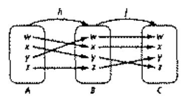
SKILLS Objective A

In 1 and 2, let $f(x) = x^2 + 7x + 2$ and $g(x) = 3x - 5$.

1. Evaluate each composite.
   a. $f(g(1))$ -8 b. $g(f(1))$ 25
2. Find a formula for each composite.
   a. $f(g(x))$ $f(g(x)) = 9x^2 - 9x - 8$ b. $g(g(x))$ $g(g(x)) = 9x - 20$
3. Let $F = \{(2, 8), (3, 5), (4, 3), (5, 2)\}$ and $G = \{(8, 2), (2, 4), (3, 3), (5, 2)\}$. Find each composite.
   a. $F \circ G$ $\{(8, 8), (2, 3), (3, 5), (5, 8)\}$ b. $G \circ F$ $\{(2, 2), (3, 2), (4, 3), (5, 4)\}$
4. Consider the functions h mapping A to B and j mapping B to C. Evaluate each composite.
   a. $h(j(w))$ x
   b. $j(h(x))$ z
   c. $(h \circ j)(z)$ w



5. Let $S: (x, y) \to (x, 2y)$ and let $T: (x - 3, y + 4)$. Write a simplified formula for $(T \circ S)(x, y)$.
   $(T \circ S)(x, y): (x, y) \to (x - 3, 2y + 4)$

PROPERTIES Objective F

6. Let $s(x) = \sqrt{x - 2}$ and $n(x) = x^2 - 1$. Give the domain of each composite.
   a. $n \circ s$ $\{x \mid x \geq 2\}$ b. $s \circ n$ $\{x \mid |x| \geq \sqrt{3}\}$

In 7 and 8, true or false. If true, justify your answer. If false, give a counterexample.

7. Let $g(t) = \frac{1}{t} - 1$. The domain of g is the same as the domain of $g \circ g$.
   false; $g(1) = 0$ but $g(g(1))$ is undefined
8. A horizontal translation of h units followed by a vertical scale change of magnitude p is the same as a vertical scale change of magnitude p followed by a horizontal translation of h units.
   false; let $T: (x, y) \to (x + 2, y)$ and $S: (x, y) \to (2x, y)$.
   $(T \circ S)(x, y) = (2x + 2, y)$ but $(S \circ T)(x, y) = (2x + 4, y)$.

Name

3-8 Lesson Master

Questions on SPUR Objectives
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SKILLS Objective B

In 1–4, a function is described. a. Give a set of ordered pairs or an equation for the inverse of the function; b. State whether the inverse is a function.

1. $y = 2 - 3x$ a. $y = \frac{2 - x}{3}$ b. function
2. $f(x) = \frac{1}{\sqrt{x}}$ a. $y = \frac{1}{x^2}$ b. function
3. $g = \{(3, 2), (2, -1), (5, 3), (3, -1)\}$ a. $\{(2, 3), (-1, 2), (3, 5), (-1, 3)\}$ b. not a function
4. $h(x) = \frac{3}{x + 4}$ a. $y = \frac{3}{x} - 4$ b. function

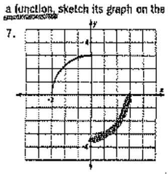
PROPERTIES Objective F

In 5 and 6, true or false. If true, explain your answer. If false, give a counterexample.

5. If a function is an even function, then its inverse is not a function.
   true; consider the even function $f(x) = x^2$, $f^{-1}(x) = \begin{cases} \sqrt{x} \\ -\sqrt{x} \end{cases}$ which is not a function
6. Given two functions, f and g, if $f(g(x)) = x$ for all x in the domain of g then f and g are inverses. false; $g(f(x)) = x$ for all x in the domain of f must also be true.

REPRESENTATIONS Objectives I, K

In 7–9, determine whether the inverse of the graphed function is a function. If the inverse is a function, sketch its graph on the same set of axes.

7.

a function

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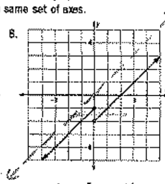
not a function

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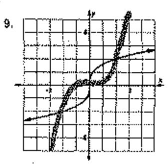
a function

$h(s(x))$
$f1(f2(x))$

Inverse Functions

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1. $f^{-1}(x) = \frac{1}{4}x + 2$ **2.** $f^{-1}(x) = \frac{1}{3}x - 2$

3. $f^{-1}(x) = 2x + 2$ **4.** $f^{-1}(x) = \frac{3}{2}x - 6$

5. $f^{-1}(x) = \frac{4}{5(x + 3)}$ **6.** $f^{-1}(x) = \sqrt[3]{x}$

Composition of Functions

Page 39

1. $3x^2 - 8$ **2.** $x^2 + 6x + 8$

3. $3x - 2$ **4.** $9x^2 - 30x + 24$

5. $3x + 4$ **6.** $3x^2 + 1$

$\sqrt{x^2 - 3} = 0$
$x^2 - 3 = 0$
$x^2 = 3$
$x = \sqrt{3}$