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Chapter 7

The Factor Theorem

Lesson 7-4 (pp. 459-464)

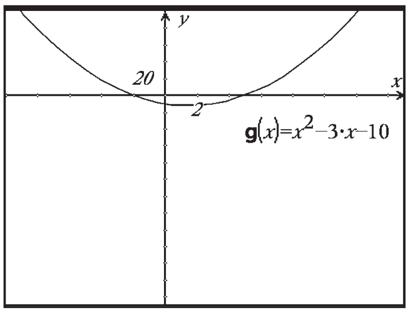
Mental Math

a. $x(x - 25) = 0$; $x = 0, 25$

b. $(x + 3)(x - 2) = 0$; $y = -3, 2$

Activity

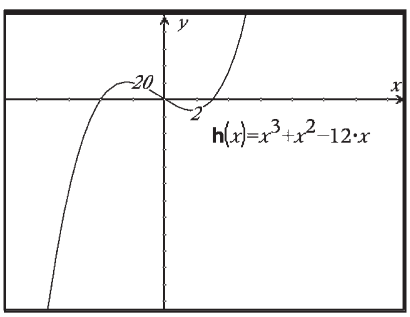
Step 1: $x = -2, 5$



Step 2: $\text{factor}(x^2-3\cdot x-10)$ $\quad (x-5)\cdot(x+2)$

Step 3: $\text{zeros}(x^2-3\cdot x-10,x)$ $\quad \{-2,5\}$

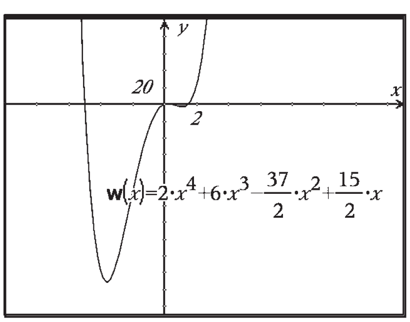
Step 4: $x = -4, 0, 3$



$\text{factor}(x^3+x^2-12\cdot x)$ $\quad x\cdot(x-3)\cdot(x+4)$

$\text{zeros}(x^3+x^2-12\cdot x,x)$ $\quad \{-4,0,3\}$

Step 5: $x = -5, 0, \frac{1}{2}, \frac{3}{2}$



$\text{factor}\left(2\cdot x^4+6\cdot x^3-\frac{37}{2}\cdot x^2+\frac{15}{2}\cdot x\right)$

$\frac{x\cdot(x+5)\cdot(2\cdot x-3)\cdot(2\cdot x-1)}{2}$

$\text{zeros}\left(2\cdot x^4+6\cdot x^3-\frac{37}{2}\cdot x^2+\frac{15}{2}\cdot x,x\right)$

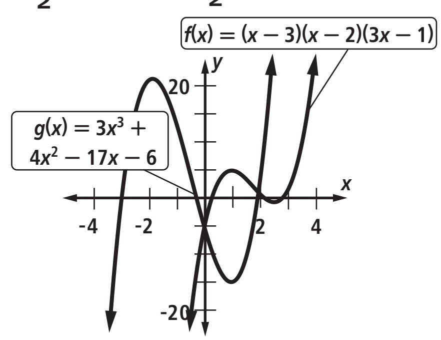
$\left\{-5,0,\frac{1}{2},\frac{3}{2}\right\}$

Step 6: Answers vary. Sample: It appears that the zeros, the x-intercepts, and the constants c from the factors $(x - c)$ of the polynomial are all equal.

Questions

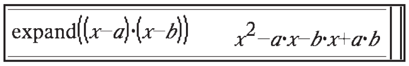
1. For a polynomial $p(x)$, $(x - c)$ is a factor of $p(x)$ if and only if $p(c) = 0$.
2. $f(17) = 0$
3. $f(x) = (x - 7)(x - 3)(x + 2)(x + 5)$
4. a. $\text{zeros}(n^3+4\cdot n^2-31\cdot n-70,n)$ $\quad \{-7,-2,5\}$

   b. $t(n) = (n + 7)(n + 2)(n - 5)$
5. Answers vary. Sample: $p(x) = x(x - 6)(x + 4) = x^3 - 2x^2 - 24x$
6. Answers vary. Sample: $p(x) = (x - 8)(x - \frac{7}{2})(x + \frac{5}{3}) = x^3 - \frac{59}{6}x^2 + \frac{53}{6}x + \frac{140}{3}$
7. Answers vary. Any two of: $(x + 2)$ is a factor of $g(x)$; $g(-2) = 0$; -2 is an x-intercept of the graph of $y = g(x)$; the remainder when $g(x)$ is divided by $(x + 2)$ is 0.
8. Answers vary. Any two of: $(x - 3)$ is a factor of $h(x)$; $h(3) = 0$; 3 is a zero of $h(x)$; the remainder when $h(x)$ is divided by $(x - 3)$ is 0.
9. Answers vary. Any two of: $(t - 2)$ is a factor of $f(t)$; $f(2) = 0$; 2 is a zero of $f(t)$; 2 is an x-intercept of the graph of $y = f(t)$.
10. $p(x) = -\frac{1}{2}x^3 - 3x^2 + \frac{9}{2}x + 27$
11. a.

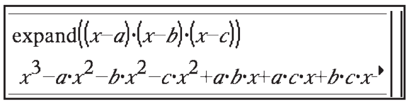


   b. No, the graphs do not appear to be the same.

12. a. $x = -3, 2$
b. –204
c. $x = -\frac{8}{3}, 7$

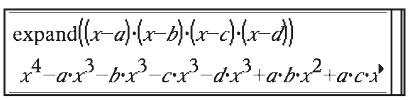
13. i. a.


b. The opposites of the zeros give coefficients for the linear terms, and the constant term is the product of the zeros.

ii. a.


Expansion: $x^3 - (a + b + c)x^2 + (ab + ac + bc)x - abc$

b. The opposites of the zeros give the coefficients for the quadratic terms; the three possible products of two zeros give coefficients for the linear terms; and the opposite of the product of all the zeros gives the constant term.

iii. a.


Expansion: $x^4 - (a + b + c + d)x^3 + (ab + ac + ad + bc + bd + cd)x^2 - (abc + abd + acd + bcd)x + abcd$

b. The opposites of the zeros give the coefficients for the cubic terms; the possible products of two zeros give coefficients for the quadratic terms; the possible products of the opposites of three zeros give coefficients for linear terms; and the product of all the zeros gives the constant term.

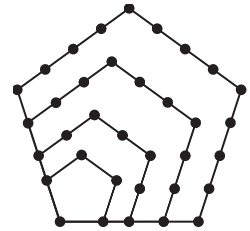
14. Answers vary. Sample:
$f(x) = -(x + 4)(x - 2)(x - 4) = -x^3 + 2x^2 + 16x - 32$

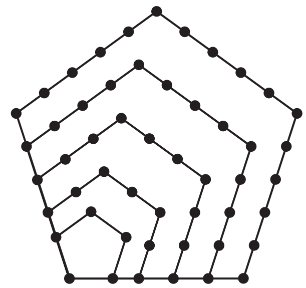
15. Answers vary. Sample:
$g(x) = (x + 8)(x)(x - 5.1)(x - 10) = x^4 - 7.1x^3 - 69.8x^2 + 408x$

16. $3x + 5$

17. $3z^2 - \frac{3}{2}z - 3$

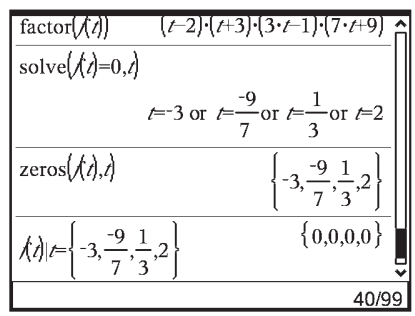
18. a. 1st differences: –12, –8, 2, 18, 40; 2nd differences: 4, 10, 16, 22; 3rd differences: 6, 6, 6; The 3rd differences are constant, so v has degree 3.
b. $v(u) = u^3 - 4u^2 - 7u + 10$

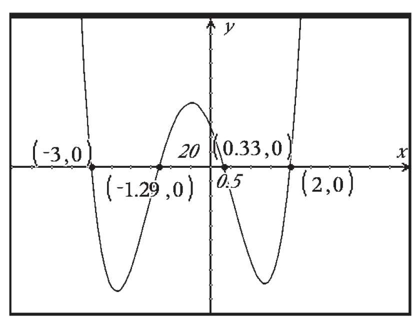
19. a. 1st differences: 4, 7, 10; 2nd differences: 3, 3; The 2nd differences are constant, so p has degree 2.
b. $p(n) = \frac{3}{2}n^2 - \frac{1}{2}n$
c.

$p(5) = 35$


$p(6) = 51$

20. a. m, p, r
b. n, q
c. $x < n, x > q$
d. $n < x < q$
e. $m < x < p, x > r$
f. $x < m, p < x < r$

21. $z = 1$

22. a. factor, solve, zeros, intersect, and such that
b.




c. Answers vary. Sample: solve, factor, and zeros are easy to use to obtain quick facts about the polynomial, but the intersect command helps visually represent the Factor-Solution-Intercept Equivalence Theorem.