Chapter 7

Complex Numbers

Lesson 7-5 (pp. 465-472)

Mental Math

a. 2 and 3

b. 3 and 4

c. 12 and 13

Guided Example 3

a. $2 - 3i$; 2; $3i$

b. $2 - 3i$; 2; $3i$; $5i + 3i$; $8i$

c. $10i$; $15i^2$; $2i$; 15

d. 2; $\frac{5}{2}$

Questions

1. $(i\sqrt{13})^2 = i^2 \cdot 13 = -1 \cdot 13 = -13$; $(-i\sqrt{13})^2 = (-i)^2 \cdot 13 = -1 \cdot 13 = -13$
2. $10i$
3. $3i\sqrt{2}$
4. –36
5. a. $3 + 2i$
   b. 13
6. a. $-9i + 7$
   b. 130
7. a. $10i$
   b. –10
8. a. cannot factor
   b. $(x + \sqrt{7})(x - \sqrt{7})$
9. a. –91
   b. $x = \frac{3}{2} + \frac{\sqrt{91}}{2}i$; $x = \frac{3}{2} - \frac{\sqrt{91}}{2}i$
10. a. –6759
    b. $x = \frac{11}{16} + \frac{3\sqrt{751}}{16}i$; $x = \frac{11}{16} - \frac{3\sqrt{751}}{16}i$
11. They are complex conjugates.
12. a. $(x - 8i)(x + 8i)$
    b. $(x - 8i)(x + 8i) = x^2 - 8ix + 8ix - 64i^2 = x^2 + 64$
13. a. $7(z - i\sqrt{2})(z + i\sqrt{2})$
    b. $7(z - i\sqrt{2})(z + i\sqrt{2}) = 7(z^2 + i\sqrt{2}z + i\sqrt{2}z - 2i^2) = 7(z^2 + 2) = 7z^2 + 14$
14. a. Answers vary. Sample: $x^2 + 225$
    b. Answers vary. Sample: $y = x^2 + 225$
15. a. False; Only complex numbers of the form $a + 0i$ are real.
    b. True; Real numbers are complex numbers of the form $a + 0i$.
16. The graph does not cross the x-axis.
17. $\frac{12 - i}{12 - i}$
18. $-5 + 3i$
19. $\frac{8}{5} - \frac{4}{5}i$
20. $7 + 3i$
21. $-115 - 236i$
22. $\frac{49}{73} + \frac{137}{73}i$
23. $i^2 = -1$; $i^3 = -i$; $i^4 = 1$; $i^5 = i$
24. $f(x) = (x - 3)(x - (1 + 5i))(x - (1 - 5i)) = x^3 - 5x^2 + 32x - 78$
25. a. $f(3 + i) = 0$; $f(3 - i) = 0$
    b. $f(x) = (x - (3 + i))(x - (3 - i)) = (x - 3 - i)(x - 3 + i)$
26. $\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2} = \frac{3 \pm \sqrt{-23}}{4} = \frac{3}{4} \pm \frac{i\sqrt{23}}{4}$, so $f(x) = 2\left(x - \left(\frac{3}{4} + \frac{i\sqrt{23}}{4}\right)\right)\left(x - \left(\frac{3}{4} - \frac{i\sqrt{23}}{4}\right)\right)$
27. a. –1
    b. –1
    c. –1
    d. –1
28. Answers vary. Any two of: $(x + 1)$ is a factor of $h(x)$; $h(-1) = 0$; –1 is a zero of $h(x)$; the remainder when $h(x)$ is divided by $(x + 1)$ is zero.
29. Answers vary. Sample:
    $f(x) = (x + 1)(x - 3)(4x + 3)$;
    $g(x) = 2(x + 1)(x - 3)(4x + 3)$;
    $h(x) = (x + 1)^2(x - 3)(4x + 3)$
30. 2
31. 2
32. $t(n) + t(n + 1) = \frac{1}{2}n(n + 1) + \frac{1}{2}(n + 1)(n + 2) = \frac{1}{2}(n + 1)(n + n + 2) = \frac{1}{2}(n + 1)(2(n + 1)) = (n + 1)^2 = s(n + 1)$.
33. a. 3; 0; 4; 2; 5; 5; 6; 9
    b. $d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$
    c. exact
    d. 1175
34. a. 1; i; 1 respectively
    b. To determine i^m, find the remainder r, when m is divided by 4. Then, $i^m = i^r$.