Back to Chapter Review

Chapter 7

Polynomial Functions

Chapter Review (pp. 496-499)

1. about –0.414
2. about 6.3
3. Because $t = 4$ is a solution to $g(t) = 0$, 4 must be a t-intercept of the graph of g.
4. about 3.16
5. about –6.2
6. Because $x = 1$ is a solution to $f(x) = 0$, 1 must be an x-intercept of the graph of f.
7. 5
8. 6
9. a. y is a polynomial of degree 2.
   b. $y = 2x^2 - 3x + 5$
10. a. y is a polynomial of degree 2.
    b. $y = \frac{3}{4}x^2 - x + 5$
11. a. $h_n = \frac{5}{2}n^2 - \frac{3}{2}n$
    b. 66178
12. Quotient: $3x^3 + 8x^2 + 6x + 7$; Remainder: 0

$$
\begin{array}{r}
3x^3 + 8x^2 + 6x + 7 \\
x - 1\overline{)3x^4 + 5x^3 - 2x^2 + x - 7} \\
\underline{3x^4 - 3x^3} \\
8x^3 - 2x^2 \\
\underline{8x^3 - 8x^2} \\
6x^2 + x \\
\underline{6x^2 - 6x} \\
7x - 7 \\
\underline{7x - 7} \\
0
\end{array}
$$

13. Quotient: $x^3 + 2x^2 + 4x + 14$; Remainder: 0

$$
\begin{array}{r}
x^3 + 2x^2 + 4x + 14 \\
x - 3\overline{)x^4 - x^3 - 2x^2 + 2x - 42} \\
\underline{x^4 - 3x^3} \\
2x^3 - 2x^2 \\
\underline{2x^3 - 6x^2} \\
4x^2 + 2x \\
\underline{4x^2 - 12x} \\
14x - 42 \\
\underline{14x - 42} \\
0
\end{array}
$$

14. Quotient: $2x^2 + \frac{10}{3}x + \frac{25}{9}$; Remainder: $\frac{280}{9}$

$$
\begin{array}{r}
2x^2 + \frac{10}{3}x + \frac{25}{9} \\
3x - 4\overline{)6x^3 + 2x^2 - 5x + 20} \\
\underline{6x^3 - 8x^2} \\
10x^2 - 5x \\
\underline{10x^2 - \frac{40}{3}x} \\
\frac{25}{3}x + 20 \\
\underline{\frac{25}{3}x - \frac{100}{9}} \\
\frac{280}{9}
\end{array}
$$

15. Quotient: $4x^3 - 4x^2 + 12x - 36$; Remainder: 225

$$
\begin{array}{r}
4x^3 - 4x^2 + 12x - 36 \\
2x + 6\overline{)8x^4 + 16x^3 + 0x^2 + 0x + 9} \\
\underline{8x^4 + 24x^3} \\
-8x^3 + 0x^2 \\
\underline{-8x^3 - 24x^2} \\
24x^2 + 0x \\
\underline{24x^2 + 72x} \\
-72x + 9 \\
\underline{-72x - 216} \\
225
\end{array}
$$

16. $x \approx -1.63, -0.74, 1.63, 0.37 \pm 0.64i$
17. $x = y$, $x \approx (-0.81 \pm 0.59i)y$, $(0.31 \pm 0.95i)y$
18. $(x - (-2 + 3i))(x - (-2 - 3i))(x - 5i)(x + 5i)$; zeros: $x = -2 \pm 3i, \pm 5i$
19. $-\sqrt{5}$
20. $x = 1$, or $x = \pm 2i$
21. Answers vary. Sample: $x^3 - 4x^2 + 6x - 4$
22. $(x - 4)(x - 2)(x - 1)(x + 1)$
23. a. $(x - \sqrt{6})(x + \sqrt{6})(x + 1)(x^2 - x + 1)$
    b. $(x - \sqrt{6})(x + \sqrt{6})(x + 1)\left(x - \frac{1 + i\sqrt{3}}{2}\right)\left(x - \frac{1 - i\sqrt{3}}{2}\right)$
24. a. $(t - 3)(t^2 + 3t + 9)$
    b. $(t - 3)\left(t + \frac{3 - 3i\sqrt{3}}{2}\right)\left(t + \frac{3 + 3i\sqrt{3}}{2}\right)$
25. a. $(x + 2)(x - 1)(x + 1)(x^2 + 1)$
    b. $(x + 2)(x - 1)(x + 1)(x - i)(x + i)$
26. $x = 8$, or $x = -4 \pm 4i\sqrt{3}$
27. $(x - 1 - i\sqrt{2})(x - 1 + i\sqrt{2})$
28. $40 - 21i$
29. $-4 + 11i$
30. $43 - 182i$

31. $\frac{101}{3} - \frac{23}{3}i$

32. $-48 - 6i$

33. $216 + 0i$

34. $2 - 3i$

35. $\frac{20}{13} - \frac{9}{13}i$

36. a. 9
    b. 5
    c. 0
    d. –17
    e. 9

37. Answers vary. Sample: $5x^3 + 2x^2 + x$

38. False; If a polynomial function p has a maximum at (–2, –5), then the graph of $y = p(x)$ never crosses the x-axis.

39. False; The roots of a polynomial function p refer to its x-intercepts.

40. False; If the graph of $y = p(x)$ has 3 x-intercepts, then $p(x)$ is degree 3 or greater.

41. a. 4
    b. –6
    c. 8

42. $f(7) = 0$

43. Answers vary. Sample: $p(x) = x^4 - 13x^3 + 53x^2 - 83x + 42$

44. Answers vary. Any two of: $(x + 11)$ is a factor of the polynomial; –11 is a zero of the polynomial; the remainder when the polynomial is divided by $(x + 11)$ is 0; –11 is an x-intercept on the graph of the polynomial.

45. Answers vary. Sample: $k(5) = 0$ and $k(x)$ has 2 complex zeros.

46. 3

47. 324

48. Answers vary. Sample: $x + 2$

49. Every real number a can be written in complex form as $a + 0i$. Its complex conjugate is $a - 0i = a$.

50. An odd degree polynomial has an odd number of zeros, counting multiplicities. Since complex zeros occur in conjugate pairs, the total number of nonreal zeros must be even. Therefore, there must be at least one real zero.

51. All four zeros must be nonreal.

52. $f(2i) = 0$ and the other zeros are $-2i$, –4, and 4.

53. True. If $2 + i$ is a zero, then $2 - i$ must also be a zero because nonreal zeros occur as conjugate pairs. If $1 + i$ were a zero, then $1 - i$ would also have to be a zero. Since the polynomial has degree 3, it only has 3 zeros, meaning that $1 + i$ cannot be a zero.

54. a. $V = \frac{1}{3}\pi r^2 h + 8r^3$
    b. $V(r) = \frac{10}{3}\pi r^2 + 8r^3$
    c. The degree of the polynomial is 3. This is reasonable because volume is a measurement of three-dimensional space.
    d. 8

55. $V(x) = 2x^3 - 600x^2 + 40000x$

56. a. $G(n) = n^2 - n$
    b. $G(28) = 756$

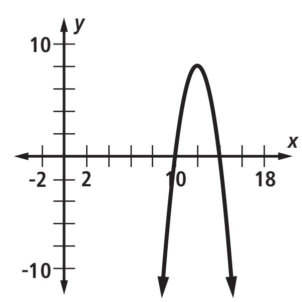
57. a. $A(x) = 350x^{13} + 350x^{12} + \cdots + 350x + 350$
    b. $\approx \$7102.40$

58. $A(\ell) = \ell(\ell + 20)$

59. $A(\ell) = \ell(\ell - 25)$

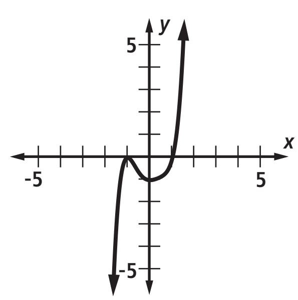
60. $\ell = w = 50$ yards; the perimeter is 200 yards; the area is 2500 square yards

61. a.



    b. Relative maximum at (12, 8)
    c. $x = 10, 14$

62. a.



    b. Relative maximum: (–1, 0)
    Relative minimum: (0, –1)
    c. $x = -1, 1$.

63. a. 3

    b. f only has 3 real zeros, which means it must be 2 nonreal zeros.

64. False. By the Number of Zeros of a Polynomial Theorem and the Factor-Solution-Intercept Equivalence Theorem, the equation has 5 complex solutions. To make the statement true, say that the equation has three real zeros, or that the equation has exactly five solutions.

65. true

66. a. $a < x < c$, $e < x < g$, $x > i$

    b. $x < a$, $c < x < e$, $g < x < i$

    c. $x < b$, $d < x < f$, $x > h$

    d. $b < x < d$, $f < x < h$