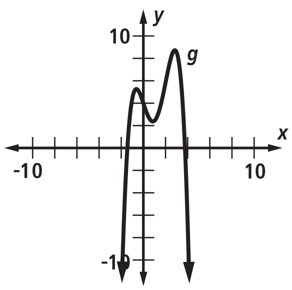
Chapter 7

Polynomial Functions

Self-Test (pp. 494-495)

1. 

2. By tracing the graph, a relative maximum point is about (–0.64, 5.31). The maximum point of the graph is about (2.81, 8.77). A relative minimum point is about (0.83, 2.42).

3. By tracing, using the intersect command on a graphing utility, or solving $g(x) = 0$, the zeros are about –1.45 and 3.72.

4. Since the largest degree of any term is 4, the polynomial is of degree 4.

5. On the graph, two real zeros can be seen where the graph crosses the axes. Therefore, there are $4 - 2 = 2$ nonreal zeros.

6.
$$\begin{array}{r} 3x^3 - 13x^2 + 29x - 59 \\ x + 2 \overline{)3x^4 - 7x^3 + 3x^2 - x - 2} \\ \underline{3x^4 + 6x^3} \\ -13x^3 + 3x^2 \\ \underline{-13x^3 - 26x^2} \\ 29x^2 - x \\ \underline{29x^2 + 58x} \\ -59x - 2 \\ \underline{-59x - 118} \\ 116 \end{array}$$

The quotient is $3x^3 - 13x^2 + 29x - 59$ and the remainder is 116.

7. A; $f(1) = 0$, so by the Factor Theorem, $(x - 1)$ is a factor of f.

8. Answers vary. Sample: Because $2 - i$ is a zero, its conjugate, $2 + i$, must also be a zero. By the Factor Theorem, the polynomial must have factors $(x - 5)$, $(x - 2 + i)$ and $(x - 2 - i)$. Multiply these to get one possible polynomial, $x^3 - 9x^2 + 25x - 25$.

9. a. $\frac{3 - 2i}{3 - 2i}$

b. $\frac{2 - 3i}{3 + 2i} = \frac{(2 - 3i)(3 - 2i)}{(3 + 2i)(3 - 2i)} = \frac{6 - 13i + 6i^2}{9 - 4i^2} = 0 - i = -i$

10. a. $2(2 + 4i) + (3 + 7i) = 7 + 15i$

b. $(2 + 4i)(3 + 7i) = 6 + 14i + 12i + 28i^2 = -22 + 26i$

11. a. The first differences are 4, 7, 10, 13, 16. The second differences are 3, 3, 3, 3. Because the 2nd differences are constant, the polynomial is of degree 2. Using quadratic regression, $P_n = \frac{3}{2}n^2 - \frac{1}{2}n$.

b. $P_{20} = \frac{3}{2} \cdot 20^2 - \frac{1}{2} \cdot 20 = 590$

12. a. $\ell + 2w = 500$, so $w = 250 - \frac{1}{2}\ell$.

b. $A = \ell w = \ell(250 - \frac{1}{2}\ell) = 250\ell - \frac{1}{2}\ell^2$

c. Because the length and width must be positive, $\ell > 0$ and $w = 250 - \frac{1}{2}\ell > 0$. Then $\ell > 0$ and $\ell < 500$, so the domain is $0 < \ell < 500$.

13. $9ab + 18b + 5a + 10 = (9ab + 18b) + (5a + 10) = 9b(a + 2) + 5(a + 2) = (9b + 5)(a + 2)$

14. Sample: $64m^6 + 27k^3 = (4m^2)^3 + (3k)^3$
$= (4m^2 + 3k)((4m^2)^2 - (4m^2)(3k) + (3k)^2)$
$= (4m^2 + 3k)(16m^4 - 12m^2k + 9k^2)$

15. Answers vary. Sample: Let $u = x^5$. Then $x^{10} - 1 = u^2 - 1 = (u - 1)(u + 1) = (x^5 - 1)(x^5 + 1) = (x - 1)(x + 1)(x^4 + x^3 + x^2 + x + 1)(x^4 - x^3 + x^2 - x + 1)$.

16. By the Polynomial Difference Theorem, the polynomial has degree 5.

17. Count the number of x-intercepts of each graph: R has 3 real zeros; S has 1 real zero; T has 1 real zero.

18. (1, 0); $R(1) = S(1) = T(1) = 0$, so by the Factor Theorem (1, 0) is a point on each graph.

19. C; S has degree 3, and because it has no nonreal zeros, by the Number of Zeros of a Polynomial Theorem, it must have 3 real zeros. Because 1 is the only x-intercept, and every real zero is an x-intercept, 1 must be a zero of multiplicity 3.

20. Because function T has degree 3 and has only 1 real zero, by the Number of Zeros of a Polynomial Theorem, it has 2 nonreal zeros.

21. There is a horizontal line that the graph of $p(x)$ appears to cross 5 times, so by the Polynomial Graph Wiggliness Theorem, $p(x)$ has degree at least 5.