

Chapter 8

How Much Does a Loan Cost?

Lesson 8-6 (pp. 536-541)

Mental Math

C

Activity

Step 1:

a.

◇	A	B	C	D
1	Month	Principal	Interest	Monthly Payment
2	1	\$6,000.00	45	\$124.55
3	2	\$5,920.45	44.4	\$124.55
4	3	\$5,840.30	43.8	\$124.55
5	4	\$5,759.56	43.2	\$124.55
6	5	\$5,678.20	42.59	\$124.55
7	6	\$5,596.24	41.97	\$124.55
8	7	\$5,513.66	41.35	\$124.55
9	8	\$5,430.46	40.73	\$124.55
10	9	\$5,346.64	40.1	\$124.55
11	10	\$5,262.19	39.47	\$124.55
12	11	\$5,177.11	38.83	\$124.55
13	12	\$5,091.39	38.19	\$124.55
14	13	\$5,005.02	37.54	\$124.55
15	14	\$4,918.01	36.89	\$124.55
16	15	\$4,830.34	36.23	\$124.55
17	16	\$4,742.02	35.57	\$124.55
18	17	\$4,653.04	34.9	\$124.55
19	18	\$4,563.38	34.23	\$124.55
20	19	\$4,473.06	33.55	\$124.55
21	20	\$4,382.06	32.87	\$124.55
22	21	\$4,290.37	32.18	\$124.55
23	22	\$4,198.00	31.49	\$124.55
24	23	\$4,104.94	30.79	\$124.55
25	24	\$4,011.17	30.08	\$124.55
26	25	\$3,916.71	29.38	\$124.55
27	26	\$3,821.53	28.66	\$124.55
28	27	\$3,725.64	27.94	\$124.55
29	28	\$3,629.04	27.22	\$124.55
30	29	\$3,531.70	26.49	\$124.55
31	30	\$3,433.64	25.75	\$124.55

◇	A	B	C	D
1	Month	Principal	Interest	Monthly Payment
32	31	\$3,334.84	25.01	\$124.55
33	32	\$3,235.31	24.26	\$124.55
34	33	\$3,135.02	23.51	\$124.55
35	34	\$3,033.98	22.75	\$124.55
36	35	\$2,932.19	21.99	\$124.55
37	36	\$2,829.63	21.22	\$124.55
38	37	\$2,726.30	20.45	\$124.55
39	38	\$2,622.20	19.67	\$124.55
40	39	\$2,517.32	18.88	\$124.55
41	40	\$2,411.65	18.09	\$124.55
42	41	\$2,305.18	17.29	\$124.55
43	42	\$2,197.92	16.48	\$124.55
44	43	\$2,089.86	15.67	\$124.55
45	44	\$1,980.98	14.86	\$124.55
46	45	\$1,871.29	14.03	\$124.55
47	46	\$1,760.77	13.21	\$124.55
48	47	\$1,649.43	12.37	\$124.55
49	48	\$1,537.25	11.53	\$124.55
50	49	\$1,424.23	10.68	\$124.55
51	50	\$1,310.36	9.83	\$124.55
52	51	\$1,195.64	8.97	\$124.55
53	52	\$1,080.05	8.1	\$124.55
54	53	\$963.60	7.23	\$124.55
55	54	\$846.28	6.35	\$124.55
56	55	\$728.08	5.46	\$124.55
57	56	\$608.99	4.57	\$124.55
58	57	\$489.01	3.67	\$124.55
59	58	\$368.12	2.76	\$124.55
60	59	\$246.34	1.85	\$124.55
61	60	\$123.63	0.93	\$124.55
62	61	\$0.00	0	

b.
$$\frac{6000(1.0075)^{60}(0.0075)}{1.0075^{60} - 1} \approx \$124.55$$

Step 2:

a. about 0.00833, or about 0.833%

b.

◇	A	B	C	D
1	Month	Principal	Interest	Monthly Payment
2	1	\$15,000.00	125.00	\$800.00
3	2	\$14,325.00	119.37	\$800.00
4	3	\$13,644.37	113.70	\$800.00
5	4	\$12,958.08	107.98	\$800.00
6	5	\$12,266.06	102.22	\$800.00
7	6	\$11,568.28	96.40	\$800.00
8	7	\$10,864.68	90.54	\$800.00
9	8	\$10,155.22	84.63	\$800.00
10	9	\$9,439.85	78.67	\$800.00
11	10	\$8,718.51	72.65	\$800.00
12	11	\$7,991.17	66.59	\$800.00
13	12	\$7,257.76	60.48	\$800.00
14	13	\$6,518.24	54.32	\$800.00
15	14	\$5,772.56	48.10	\$800.00
16	15	\$5,020.66	41.84	\$800.00
17	16	\$4,262.50	35.52	\$800.00
18	17	\$3,498.02	29.15	\$800.00
19	18	\$2,727.17	22.73	\$800.00
20	19	\$1,949.90	16.25	\$800.00
21	20	\$1,166.15	9.72	\$800.00
22	21	\$375.87	3.13	\$379.00
23	22	\$0.00	0	

c.

◇	A	B	C	D
1	Month	Principal	Interest	Monthly Payment
2	1	\$15,000.00	125.00	\$400.00
3	2	\$14,725.00	122.71	\$400.00
4	3	\$14,447.71	120.40	\$400.00
5	4	\$14,168.11	118.07	\$400.00
6	5	\$13,886.17	115.72	\$400.00
7	6	\$13,601.89	113.35	\$400.00
8	7	\$13,315.24	110.96	\$400.00
9	8	\$13,026.20	108.55	\$400.00
10	9	\$12,734.75	106.12	\$400.00
11	10	\$12,440.88	103.67	\$400.00
12	11	\$12,144.55	101.20	\$400.00
13	12	\$11,845.75	98.71	\$400.00
14	13	\$11,544.47	96.20	\$400.00
15	14	\$11,240.67	93.67	\$400.00
16	15	\$10,934.34	91.12	\$400.00
17	16	\$10,625.46	88.55	\$400.00
18	17	\$10,314.01	85.95	\$400.00
19	18	\$9,999.96	83.33	\$400.00
20	19	\$9,683.29	80.69	\$400.00

◇	A	B	C	D
1	Month	Principal	Interest	Monthly Payment
21	20	\$9,363.99	78.03	\$400.00
22	21	\$9,042.02	75.35	\$400.00
23	22	\$8,717.37	72.64	\$400.00
24	23	\$8,390.02	69.92	\$400.00
25	24	\$8,059.93	67.17	\$400.00
26	25	\$7,727.10	64.39	\$400.00
27	26	\$7,391.49	61.60	\$400.00
28	27	\$7,053.09	58.78	\$400.00
29	28	\$6,711.86	55.93	\$400.00
30	29	\$6,367.79	53.06	\$400.00
31	30	\$6,020.86	50.17	\$400.00
32	31	\$5,671.03	47.26	\$400.00
33	32	\$5,318.29	44.32	\$400.00
34	33	\$4,962.61	41.36	\$400.00
35	34	\$4,603.97	38.37	\$400.00
36	35	\$4,242.33	35.35	\$400.00
37	36	\$3,877.69	32.31	\$400.00
38	37	\$3,510.00	29.25	\$400.00
39				

Step 3:

◇	A	B	C	D
1	Month	Principal	Interest	Monthly Payment
2	1	\$15,000.00	125.00	\$484.00
3	2	\$14,641.00	122.01	\$484.00
4	3	\$14,279.01	118.99	\$484.00
5	4	\$13,914.00	115.95	\$484.00
6	5	\$13,545.95	112.88	\$484.00
7	6	\$13,174.83	109.79	\$484.00
8	7	\$12,800.62	106.67	\$484.00
9	8	\$12,423.30	103.53	\$484.00
10	9	\$12,042.82	100.36	\$484.00
11	10	\$11,659.18	97.16	\$484.00
12	11	\$11,272.34	93.94	\$484.00
13	12	\$10,882.28	90.69	\$484.00
14	13	\$10,488.96	87.41	\$484.00
15	14	\$10,092.37	84.10	\$484.00
16	15	\$9,692.47	80.77	\$484.00
17	16	\$9,289.24	77.41	\$484.00
18	17	\$8,882.65	74.02	\$484.00
19	18	\$8,472.68	70.61	\$484.00
20	19	\$8,059.28	67.16	\$484.00

◇	A	B	C	D
1	Month	Principal	Interest	Monthly Payment
21	20	\$7,642.44	63.69	\$484.00
22	21	\$7,222.13	60.18	\$484.00
23	22	\$6,798.31	56.65	\$484.00
24	23	\$6,370.97	53.09	\$484.00
25	24	\$5,940.06	49.50	\$484.00
26	25	\$5,505.56	45.88	\$484.00
27	26	\$5,067.44	42.23	\$484.00
28	27	\$4,625.67	38.55	\$484.00
29	28	\$4,180.21	34.84	\$484.00
30	29	\$3,731.05	31.09	\$484.00
31	30	\$3,278.14	27.32	\$484.00
32	31	\$2,821.46	23.51	\$484.00
33	32	\$2,360.97	19.67	\$484.00
34	33	\$1,896.64	15.81	\$484.00
35	34	\$1,428.45	11.90	\$484.00
36	35	\$956.35	7.97	\$484.00
37	36	\$480.32	4.00	\$484.33
38	37	\$0.00		
39				

Note that the last payment adds the remaining 33 cents that would have been left after the 36th monthly payment.

Step 4:

$$\frac{15000(1.008333)^{36}(0.008333)}{(1.008333)^{36} - 1} \approx \$484$$

Guided Example

0.4875%; 0.004875; 240; \$250,000; 250,000; 0.004875; 240; 0.004875; 240

Questions

- 36 months (3 years)
 - \$2500
 - about 0.667%
- M = monthly payment; P = principal; r = monthly interest rate, n = term of the loan in months

 - about \$3,175.59
 - about \$78.34
- the total value of the first monthly payment after 48 months
 - $M(1.0075)^{47}$; $(1.0075)^{-1}$
 - M ; 1.0075
- about \$1,117.65

 - about \$1,687.71
 - about \$843.86
 - equal to half of the answer to Part a; Replacing P with $\frac{1}{2}P$ in the payment formula halves the final answer.
- about \$2,202.13
 - about \$1,619.35
 - greater than half of the answer to Part a; The monthly payment is not directly proportional to the interest rate.
- $M = \frac{P(1+r)^n r}{(1+r)^n - 1}$, so $\frac{2P(1+r)^n r}{(1+r)^n - 1} = 2\left(\frac{P(1+r)^n r}{(1+r)^n - 1}\right) = 2M$.

 - 4 years; \$172.98
 - 3 years; \$1,167.24
- 1, 11, 121, 1331, 14641
 - $1 + 11 + 121 + 1331 + 14641 = 16105$
- 5
- $\frac{341}{16} \approx 21.3$
- $\frac{242461}{8192} \approx 29.6$
- 49 terms
 - $S_{49} = \frac{49}{2}(87 - 729)$
 - 15,729
- false
- $\frac{1 - \sqrt{3}}{2}$
- 1
- Answers vary. Sample: With an interest rate of 5.44% on a 5-year loan with principal \$37,200 (on a car costing \$46,500), the monthly payment would be about \$709.53.