# §9-5 Log Properties

1. Multiplication Property: $\log_b MN = \log_b M + \log_b N$

2. Quotient Property: $\log_b \dfrac{M}{N} = \log_b M - \log_b N$

3. Power to a Power Property: $\log_b M^n = n \log_b M$

If you are given: $\log_{10} 4 = .6021$ and $\log_{10} 6 = .7782$, use the Laws and the given to find the following. Justify each step with the properties listed above or basic operations property.

Example $\log_{10} 24$

$\qquad \log_{10}(4 \cdot 6)$     Factors of 24

$\qquad \log_{10} 4 + \log_{10} 6$   Multiplication Property

$\qquad .6021 + .7782$     Substitution Property

$\qquad 1.3803$       Addition

2. $\log_{10} 16$    3. $\log_{10} \dfrac{3}{2}$    4. $\log_{10} \dfrac{1}{4}$    5. $\log_{10} 36$

6. $\log_{10} \sqrt{6}$    7. $\log_{10} 2$   (hint: $2 = \sqrt{4}$)    8. $\log_{10}\left(\dfrac{1}{16}\right)$

Even though we were only given $\log_{10} 4$ and $\log_{10} 6$ we know $\log_{10} 10 = 1$ and $\log_{10} 100 = 2$

9. $\log_{10} 40$      10. $\log_{10} 400$

In the preceding problems we had to work with decimal values. The following problems involve the same 3 laws of logarithms, but we will use variables instead of decimals.

Given: $\log_2 9 = c$ and $\log_2 10 = d$

Find the following in terms of $c$ and $d$

11. $\log_2 90 =$    12. $\log_2 81 =$    13. $\log_2\left(\dfrac{10}{9}\right) =$

14. $\log_2 \sqrt{10} =$    15. $\log_2\left(\dfrac{1}{9}\right) =$    16. $\log_2\left(\dfrac{1}{10}\right)$

17. $\log_2 3$    18. $\log_2 900 =$    19. $\log_2\left(\sqrt[3]{9}\right) =$

20. You were given the $\log_2 9$ and $\log_2 10$, but you also know $\log_2 2 = 1$, use this to find

$\qquad \log_2 18 =$

Select the correct multiple choice:

21. $\log xy^2 =$

  a) $2 \log xy$    b) $2 \log x + \log y$   c) $2 \log x + 2 \log y$   d) $\log x + 2 \log y$

22. $\log x \cdot \log y =$

    a) $\log (x + y)$      b) $\log (x \cdot y)$      c) $\log x + \log y$      d) none of these

23. $\log x - \log y =$

    a) $\log \dfrac{x}{y}$      b) $\dfrac{\log x}{\log y}$      c) both 'a' and 'b'      d) neither 'a' or 'b'

24. $\log 100^{4x} =$

    a) $4x$      b) $6x$      c) $8x$      d) $16x$

25. $\log 2x =$

    a. $\log 2 + \log x$      b. $\log 2 \cdot \log x$      c. $2 + x$      d. $2x$

26. $\log \sqrt{3} =$

    a. $\log \left(\dfrac{1}{2} \cdot \sqrt{3}\right)$      b. $\log \dfrac{3}{2}$      c. $\dfrac{1}{2} \log \sqrt{3}$      d. $\dfrac{1}{2} \log 3$

27. $\log x + \log y + \log z =$

    a. $\log (x + y + z)$      b. $\log (x \cdot y \cdot z)$      c. $\log x \cdot \log y \cdot \log z$

28. $\log_x (x^w) =$

    a. $\log w$      b. $\log x^w$      c. $w$      d. $x^w$

29. Which student solved for x correctly in the following problem?    $2 \log x = 4$

| Alice | Bob | Carl | David |
|---|---|---|---|
| $2 \log x = 4$ | $2 \log x = 4$ | $2 \log x = 4$ | $2 \log x = 4$ |
| $\log x^2 = 4$ | $\log x^2 = 4$ | $\log x^2 = 4$ | $\log x^2 = 4$ |
| $x^2 = 4$ | $x^2 = 4$ | $x^2 = 10^4$ | $x^2 = 10^4$ |
| $x = 2$ | $x = \pm 2$ | $x^2 = 10000$ | $x^2 = 10000$ |
| | | $x = 100$ | $x = \pm 100$ |

30. Which student solved for x correctly in the following problem?    $2 \log 3 + \log x = \log 36$

**Astro**

$2 \log 3 + \log x = \log 36$

$\log 9 + \log x = \log 36$

$\log 9x = \log 36$

$9x = 36$

$x = 4$

**Bella**

$2 \log 3 + \log x = \log 36$

$\log 9 + \log x = \log 36$

$\log (9 + x) = \log 36$

$9 + x = 36$

$x = 27$

**Chu**

$2 \log 3 + \log x = \log 36$

$2(\log 3 + \log x) = \log 36$

$2 \log 3x = \log 36$

$\log 3x^2 = \log 36$

$3x^2 = 36$

$x^2 = 12$

$x = \sqrt{12}$

**Domingo**

$2 \log 3 + \log x = \log 36$

$2(\log 3 + \log x) = \log 36$

$2 \log 3x = \log 36$

$\log (3x)^2 = \log 36$

$9x^2 = 36$

$x^2 = 4$

$x = 2$

$\log_3 25$ becomes $3^x = 25$     Between what 2 consecutive integers will x lie?

$$3^2 = 9$$
$$3^x = 25$$
$$3^3 = 27$$

So x is between 2 and 3.    Would it be closer to 2 or closer to 3? _____

Determine which two integers the following logarithms lie between:

1.    $\log_2 30$         2.    $\log_7 9$         3.    $\log_4 100$

4.    $\log_3 200$        5.    $\log_{10} 7500$

Rewrite the following using the change of base formula. Change into the indicated base.

6.    $\log_5 7$   to base 2          7.    $\log_9 4$   to base 6

8.    $\log_2 3$   to base 10        9.    $\log_8 5$   to base e

Express the following as a single log:    You can use the change of base formula

10.   $\dfrac{\log_5 8}{\log_5 7}$      11.   $\dfrac{\log_9 12}{\log_9 4}$      12.   $\dfrac{\log_2 6}{\log_2 10}$      13.   $\dfrac{\log 11}{\log 5}$      14.   $\dfrac{\ln 4}{\ln 3}$

Express the following as a single log. Then simplify the final answer.

15.   $\dfrac{\log_4 49}{\log_4 7}$         16.   $\dfrac{\log_8 81}{\log_8 3}$         17.   $\dfrac{\log 64}{\log 4}$

18.   $\dfrac{\log_5 2}{\log_5 8}$          19.   $\dfrac{\log \sqrt{2}}{\log 2}$         20.   $\dfrac{\ln 32}{\ln 2}$

21.   $\log_5 7 =$

    a.   $\log 5 - \log 7$     b.   $\log 7 - \log 5$     c.   $7 \cdot \log 5$            d.   $\dfrac{\log 7}{\log 5}$

22.   $\log_8 20 =$

    a.   $\dfrac{\log_3 20}{\log_3 8}$        b.   $\log\left(\dfrac{20}{8}\right)$     c.   $\log 20 - \log 8$     d.   $20 \log 8$

23.   $\dfrac{\log_7 16}{\log_7 8} =$

    a.   $\log_7 16 - \log_7 8$     b.   $\log_8 16$     c.   $\log 2$     d.   $2$

# §9-5 Log Properties - Key

② 1.2042     ⑪ c+d    ⑳ c+1    ㉙ ~~David~~ Carl

③ 0.1761    ⑫ 2c    ㉑ d    ㉚ Astro

④ -0.6021    ⑬ d-c    ㉒ d

⑤ 1.5564    ⑭ $\frac{1}{2}$d    ㉓ a

⑥ 0.3891    ⑮ -c    ㉔ c

⑦ 0.30105    ⑯ -d    ㉕ a

⑧ -1.2042    ⑰ $\frac{1}{2}$c    ㉖ d

⑨ 1.6021    ⑱ c+2d    ㉗ b

⑩ 2.6021    ⑲ $\frac{1}{3}$c    ㉘ c

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① 4+5    ⑪ $\log_4 12$    ㉑ d

② 1+2    ⑫ $\log 6$    ㉒ a

③ 3+4    ⑬ $\log_5 11$    ㉓ b

④ 4+5    ⑭ $\log_3 4$

⑤ 3+4    ⑮ 2

⑥ $\dfrac{\log_2 7}{\log_2 5}$    ⑯ 4

⑦ $\dfrac{\log_6 4}{\log_6 9}$    ⑰ 3

⑧ $\dfrac{\log 3}{\log 2}$    ⑱ $\frac{1}{3}$

     ⑲ $\frac{1}{2}$

     ⑳ 5

⑨ $\dfrac{\ln 5}{\ln 8}$

⑩ $\log_7 8$